Optimization of Integrated Reverse Logistics Networks with Different Product Recovery Routes

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Abstract

The awareness of importance of product recovery has grown swiftly in the past few decades. This paper focuses on a problem of inventory control and production planning optimization of a generic type of an integrated Reverse Logistics (RL) network which consists of a traditional forward production route, two alternative recovery routes, including repair and remanufacturing and a disposal route. It is assumed that demand and return quantities are uncertain. A quality level is assigned to each of the returned products. Due to uncertainty in the return quantity, quantity of returned products of a certain quality level is uncertain too. The uncertainties are modelled using fuzzy trapezoidal numbers. Quality thresholds are used to segregate the returned products into repair, remanufacturing or disposal routes. A two phase fuzzy mixed integer optimization algorithm is developed to provide a solution to the inventory control and production planning problem. In Phase 1, uncertainties in quantity of product returns and quality of returns are considered to calculate the quantities to be sent to different recovery routes. These outputs are inputs into Phase 2 which generates decisions on component procurement, production, repair and disassembly. Finally, numerical experiments and sensitivity analysis are carried out to better understand the effects of quality of returns and RL network parameters on the network performance.

Keywords: Supply Chain Management, Reverse Logistics, Quality of Returned Products, Uncertainty Modelling, Inventory Control, Fuzzy Optimization.

1. Introduction

Within the past few decades, environmental concerns have led to a significant increase in product recovery activities and interest in sustainability of supply chains and logistics networks. Consumer's inclination toward 'green logistics', legal pressure and possible economic benefit are among the main reasons which led manufacturers to integrate recovery

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activities into their processes (Ilgin and Gupta, 2010). RL concerns handling of the flow of material and production from the point of consumption to the point of origin (Fleischmann et al., 1997). It covers product recovery activities which are crucial to sustainability, such as repair, remanufacturing and recycling. While recycling typically refers only to the reuse of materials used for a product without preserving its structure, repair usually involves activities necessary to restore a damaged product into the working order, while preserving its integrity. In contrast, remanufacturing comprises disassembly, replacement of components where necessary and assembly of a product to bring it back into as-good-as-new condition.

One of the most important features of the reverse flow is the presence of uncertainty in both quantity and quality of returned products which needs to be considered when developing quantitative models of reverse flows (Inderfurth, 2005; Fleischmann et al., 1997). Quality of returned products has been discussed in the literature from various point of view such as inventory control, buy-back price, different markets for new and repaired products, and so on (Dobos and Richter, 2006; Zikopoulos and Tagaras, 2007; Aras et al., 2004; Mitra, 2007).

The focus of this paper is on RL networks with two alternative recovery routes, including repair and remanufacturing, which are integrated with a traditional forward production route and a disposal option. Return products are inspected to determine their quality. They are separated into repair, remanufacturing and disposal routes based on repair and remanufacturing quality thresholds. The effects of different repair and remanufacturing thresholds on the RL network performance are examined.

In this paper, fuzzy sets are used to describe uncertainty in both demand and quantity of returned products of a specific quality level. One of the main advantages that fuzzy sets provide is the possibility of describing parameters as linguistic variables (Zadeh, 1975). In this approach, in the absence of statistical data, the expert can give linguistic descriptions of the quantity values which are modelled using fuzzy numbers, for example, returned quantity is 'considerably more than x', 'about x', 'more than x but less than y', etc. (Petrovic et al., 2008).

This paper is arranged as follows. Section 2 will briefly introduce the relevant literature. In Section 3, the problem statement is presented by describing RL networks under consideration and the main assumptions made. In Section 4, a fuzzy mixed integer optimization model of the RL network is presented. Using the model described in Section 4, a set of numerical experiments are conducted and the results are reported in Section 5. Finally, in Section 6 the paper is concluded by discussing the outcomes and possible future directions.

2. Literature Review

In the past few decades, various mathematical models for RL network design, distribution, inventory control and production planning have been proposed in the literature (Ilgin and Gupta, 2010; Faccio et al., 2014). Here, we focus on the literature on RL which consider the quality of returned products only. Various approaches have been proposed to deal with the quality of returned products and inherent uncertainty. One of the common approaches is to model the quality by a probabilistic yield rate which specifies the probability of a single product being successfully recovered. In this approach, only two outcomes are considered:

either a returned product is recoverable or it is not. Using yield rates, Dobos and Richter (2006) analysed the case of lot-sizing in a production and recovery environment with two options: either to buyback all returned items from the supplier and use the ones which are recoverable or to buyback the recoverable products only. Inderfurth (2005) developed an optimization model for an integrated RL system with stationary demand, equal lead times and stochastic uncertainty in both return quantity and quality. Zikopoulos and Tagaras (2007) considered a case of two alternative collection points with different, but probabilistically correlated yield rates considering a single time period. Furthermore, Mukhopadhyay and Ma (2009) investigated yield rate of returned products in relation with production/recovery activities. Different scenarios were investigated with respect to when and how much information about yield rate was available. Similarly, Yoo et al. (2012) considered a value of information in lot sizing decisions for a single period production/recovery network when two recovery options were available and the inspection process was imperfect, but could be improved at a cost. In addition, Nenes et al. (2010) compared several alternative policies for production planning in the presence of returned products with either as-good-as-new or remanufacturable quality levels. Moreover, El Saadany and Jaber (2010) extended the model by Dobos and Richter (2006) including the return rate as a function of purchasing price and acceptance quality level.

Another approach proposed to handle quality of returned products has been to assume a set of predefined quality levels that have different acquisition costs, remanufacturing costs and lead times. Depending on these parameters, a particular quality level is specified to be desirable for certain recovery activities. Aras et al. (2004) used a Markov chain based model to show the advantage of prioritizing returned products for recovery based on their quality. Behret and Korugan (2009) analysed an integrated manufacturing/remanufacturing system in which returned products are inspected and then classified into three quality levels, namely bad, average and good, where each level can be recovered using its own recovery facility with the corresponding recovery cost and time, or disposed.

Jayaraman (2006) proposed a linear programming model for production planning in a closed-loop RL network with predefined quality levels and zero lead times. Additionally, Das and Chowdhury (2012) utilised an MIP model for RL production planning with product design decisions and quality considerations. Mahapatra et al. (2012) also examined the effect of heterogeneous quality of return and non-uniform quantity of return in integrated RL networks using an MILP model. In the similar line of research, Nenes and Nikolaidis (2012) proposed an MILP based multi period model with deterministic demand and return quantities. They assumed that 3rd party collection sites had batches of returned products available which the recovery facility might choose to acquire or ignore. Furthermore, it had the option of using a certain part of acquired batches. In this model, the quantity of products which belong to a certain quality level for each particular batch was known. Additionally, Das and Dutta (2013) used system dynamics in an integrated reverse network with three recovery options: repair, remanufacturing and recycling. Quality of return was modelled as fixed percentages of products which could go to each recovery route. However, simulation of network behaviour using a custom policy without setup costs was the focus of this work.

Furthermore, Guo et al. (2014) proposed a network with two recovery routes: disassembly and repair where each route satisfied a separate demand. Uncertainty was taken into account by using stochastic parameters but quality of return, variations in demand and return, setup costs and lead times were not considered.

Alternatively, Galbreth and Blackburn (2006) explored the possibility of using a threshold quality level to determine products which were acceptable for the recovery activity. Remanufacturing costs was assumed to be a continuous function of quality and both the acceptable quality threshold and the total return rate were determined in such a way as to minimise procurement and remanufacturing costs in a single period setting.

Most of the RL models, which include quality of return, consider a single recovery route only (for example, Nenes and Nikolaidis (2012) and Das and Chowdhury (2012)). Additionally, some authors included alternative recovery options such as different facilities for the same type of recovery (Souza et al., 2002; Behret and Korugan, 2009). However, different types of recovery such as repair and remanufacturing have fundamental differences which lead to considerably different network structures. For example, Jayaraman (2006) considered an optimization model for an RL network with reuse and remanufacturing options. The author assumed a zero lead time with deterministic demand and return. Guide et al. (2005) considered a recovery network with repair and refurbishing options, deterministic demand and a simple yield rate based quality model. Similarly, Mitra (2007) analysed a single period recovery network with remanufacturing and refurbishing, without uncertainty and zero lead time.

We propose a novel multi period, multi quality level, multi recovery route optimisation model with different lead times along a RL network and uncertainty in demand, return quantities and return qualities. Quality thresholds, which determine the recovery route that returned products should follow, are handled in the model. Numerical experiments and sensitivity analysis carried out contribute to better understanding of the impact of relevant RL network parameters on the optimal quality thresholds and on the network performance. The focus is placed on the following network parameters: quantity of returned products, unit repair costs, unit production cost, setup costs and unit disposal cost. Their impact on the network performance is quantitatively analysed.

3. Problem Statement

An RL network with two possible recovery routes, including repair and remanufacturing, disposal route, as well as a main production/forward logistics route is considered. Remanufacturing route comprises disassembly of returned products, the stock of the disassembled components in the component inventory and subsequent production. Both repaired and remanufactured products are stored in the final products inventory assuming their as-good-as-new condition. Quality inspection, carried out for each returned product, determines the appropriate route that the return should take. In addition, the final products inventory is replenished by the standard forward production route which utilises new components purchase. The RL network is presented in Figure 1.

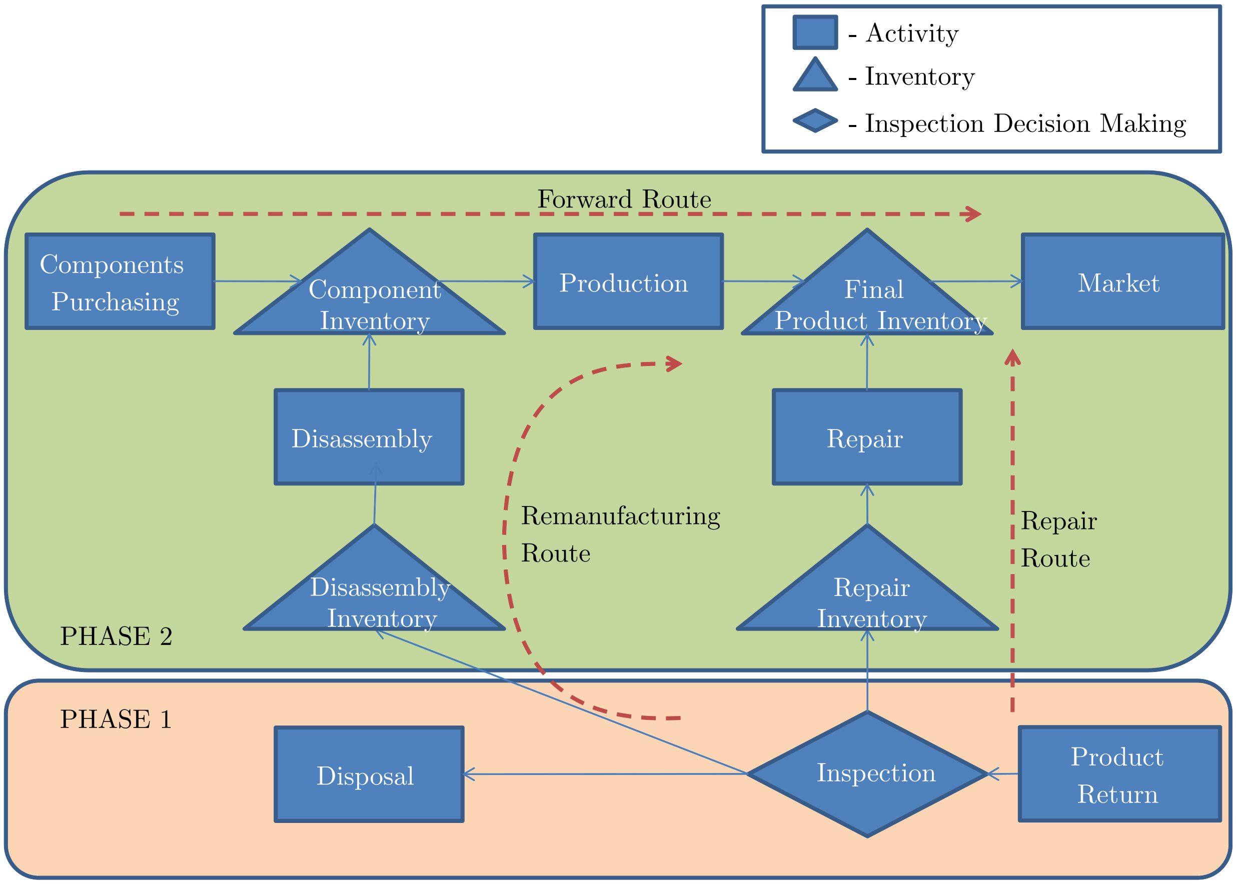

Figure 1: Diagram of the integrated RL network

It is assumed that a product in the RL network includes a single recoverable component. It is worth noting that the product can include more than one component, but it is assumed that only one component is recoverable in the remanufacturing route. An example of such a product is tyre (Lebreton and Tuma, 2006). Multiple tyre recovery options are available such as re-grooving (i.e., repair) and rethreading (i.e., remanufacturing). In this case, a tyre casing can be considered as the single recoverable component which can also be procured from external sources and used in the forward route. Burning the tyres (i.e., disposal) is used when the quality of returned tyres are not satisfactory for recovery.

Economic efficiencies of recovery routes are dependent on the quality of returned products. Typically, repair is more efficient for products of relatively good quality, while remanufacturing is more appropriate for relatively more defective/damaged products. A returned product is assigned an ordinal quality level which leads to different repair and remanufacturing costs; a higher quality level incurs cheaper repair and remanufacturing costs.

The following assumptions are made:

- The RL network is evaluated using the cost function only. The cost includes inventory holding costs, production and recovery (i.e. repair and disassembly) variable unit costs, production and recovery (i.e. repair and disassembly) setup costs and lost sale costs.
- The RL network is considered within a time horizon.
- The network is dynamic; production and recovery activities have different lead times.
- A single product type consisting of a single recoverable component is considered.
- Recovered products, both repaired and remanufactured, are considered as-good-as-new.
- Returned products are inspected on a first come first served basis (i.e. it is not possible to prioritise inspection of some products over the others).
- Demand and return quantities are not precisely known, and they are specified using fuzzy numbers.
- The appropriate recovery path is assigned based on quality thresholds.

4. RL Optimization Model

The complex structure of the RL network considered, along with different lead times of different routes, setup costs, impact of quality of returned products on the economic efficiency and uncertainty in demand and returned products quantities of different qualities make the optimization of the whole network a difficult task. In the model proposed, the RL network is split into two sub-networks which are considered in two phases. Phase 1 considers inspection and the disposal route, while Phase 2 considers the rest of the network including repair and disassembly inventories and their respective activities, as well as the forward route including procurement, components inventory, production and final products inventory.

Fuzzy return quantities of different qualities are inputs into Phase 1 which calculates the fuzzy quantities of products to be sent to the repair, remanufacturing and disposal routes. Based on these inputs from Phase 1, and fuzzy demand, the optimization model of Phase 2 determines quantities to be repaired, disassembled, new components to be procured and final products to be produced in each period of time within the time horizon under consideration.

4.1. Phase 1

In this model, the basic policy of inspection of returned products is considered where they are inspected promptly upon arrival. In order to determine which route each returned product should take, the recovery or disposal route, Phase 1 uses quality thresholds to separate the returned products into disposable, remanufacturable and repairable products.

The following notations are used:

Table 1: Notations used in Phase 1

$t \in \{1, 2, ..., T\}$	Time periods within the time horizon under consideration.
$q \in \{1, 2, ..., Q\}$	Quality levels.
$\widetilde{BI}(t, q)$	Fuzzy quantity of returned products at period t of quality level q, represented as trapezoidal membership function $(\underline{BI}(t, q), BI_L(t, q), BI_U(t, q), \overline{BI}(t, q))$.
$c_R(q)$	Unit cost of repair of product of quality level q.
$c_M(q)$	Unit cost of disassembly of product of quality level q.
c_G	Unit cost of disposal.
QT_R	Quality threshold for returned products acceptable for repair.
QT_M	Quality threshold for returned products acceptable for remanufacturing.
$\widetilde{B'_R}(t, q)$	Fuzzy quantity of inspected products of quality level q to be sent to the repair route at period t, represented as trapezoidal membership function $(\underline{B'_R}(t, q), B'_{RL}(t, q), B'_{RU}(t, q), \overline{B'_R}(t, q))$.
$\widetilde{B'_M}(t, q)$	Fuzzy quantity of inspected products of quality level q to be sent to the remanufacturing route at period t, represented as trapezoidal membership function $(\underline{B'_M}(t, q), B'_{ML}(t, q), B'_{MU}(t, q), \overline{B'_M}(t, q))$.
$\widetilde{B_R}(t)$	Total fuzzy quantity of inspected products to be sent to the repair route at period t, represented as trapezoidal membership function $(\underline{B_R}(t), B_{RL}(t), B_{RU}(t), \overline{B_R}(t))$.
$\widetilde{B_M}(t)$	Total fuzzy quantity of inspected products to be sent to the remanufacturing route at period t, represented as trapezoidal membership function $(\underline{B_M}(t), B_{ML}(t), B_{MU}(t), \overline{B_M}(t))$.
$\widetilde{B_G}(t)$	Fuzzy quantity of inspected products to be sent to the disposal route at period t, represented as trapezoidal membership function $(\underline{B_G}(t), B_{GL}(t), B_{GU}(t), \overline{B_G}(t))$.
$c_{avg,R}$	Average cost of repair per product with respect to different returned products qualities.
$c_{avg,M}$	Average cost of disassembly per product with respect to different returned products qualities.
w_0	Total disposal cost.

Quality levels assigned to returned products after inspection are discrete and crisp values from 1 to Q, where 1 represents the lowest, while Q represents the highest quality levels. Two thresholds, including remanufacturing and repair thresholds, divide the quality range into three quality groups: repairable, remanufacturable and disposable products, as shown in Figure 2. It is assumed that the thresholds are determined in advance.

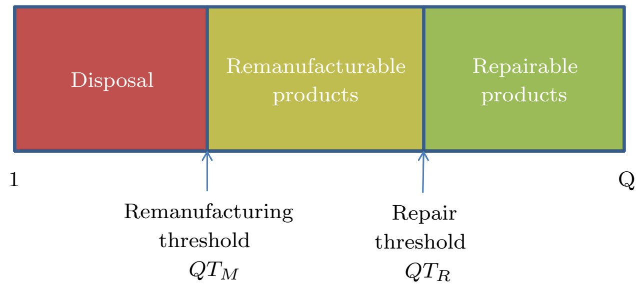

Figure 2: Quality groups determined by two quality thresholds

As the quantities of returned products of different quality levels are fuzzy, quantities to be sent to the repair, remanufacturing and disposal routes during the time horizon under consideration become fuzzy too. The following formulas are used to determine these fuzzy quantities using fuzzy operators given in Appendix A:

$$\widetilde{B'_R}(t,q) = \begin{cases} \widetilde{BI}(t,q) & QT_R \leq q \leq Q \\ 0 & \text{otherwise} \end{cases} \quad \text{for all } q \in \{1,2,...,Q\}$$

$$\widetilde{B'_M}(t,q) = \begin{cases} \widetilde{BI}(t,q) & QT_M \leq q < QT_R \\ 0 & \text{otherwise} \end{cases} \quad \text{for all } q \in \{1,2,...,Q\}$$

$$\widetilde{B_R}(t) = \sum_{q=1}^{Q} \widetilde{B'_R}(t,q)$$

$$\widetilde{B_M}(t) = \sum_{q=1}^{Q} \widetilde{B'_M}(t,q)$$

$$\widetilde{B_G}(t) = \sum_{q=1}^{QT_M-1} \widetilde{BI}(t,q)$$

The returned fuzzy quantities of products to be repaired, remanufactured or disposed incur the following costs:

$$c_{avg,R} = \frac{\sum_{q=1}^{Q} c_R(q) \sum_{t=1}^{T} Defuzz(\widetilde{B'_R}(t,q))}{\sum_{q=1}^{Q}\sum_{t=1}^{T} Defuzz(\widetilde{B'_R}(t,q))}$$

$$c_{avg,M} = \frac{\sum_{q=1}^{Q} c_M(q) \sum_{t=1}^{T} Defuzz(\widetilde{B'_M}(t,q))}{\sum_{q=1}^{Q}\sum_{t=1}^{T} Defuzz(\widetilde{B'_M}(t,q))}$$

$$w_0 = c_G \sum_{t=1}^{T} Defuzz(\widetilde{B_G}(t))$$

where the operator $Defuzz$ represents defuzzification of a fuzzy set, defined in Appendix A.

4.2. Phase 2

A fuzzy mixed integer programming model which accommodates uncertainty in demand and quantity of products sent for repair and remanufacturing is proposed.

The following notations are used:

Table 2: Notations used in Phase 2

$t \in \{1, 2, ..., T\}$	Time periods.
$\widetilde{D}(t)$	Fuzzy quantity of demand at period t, represented as trapezoidal membership function $(\underline{D}(t), D_L(t), D_U(t), \overline{D}(t))$.
$\widetilde{B_R}(t)$	Fuzzy quantity of products sent to repair at period t (calculated in Phase 1).
$\widetilde{B_M}(t)$	Fuzzy quantity of products sent to remanufacturing at period t (calculated in Phase 1).
LT_C	Lead time of procurement.
LT_P	Production time.
LT_R	Repair time.
LT_M	Disassembly time.
h_S	Unit holding costs of the final products per unit time period.
h_C	Unit holding costs of the components per unit time period.
h_R	Unit holding costs of the repair per unit time period.
h_M	Unit holding costs of the disassembly per unit time period.
c_C	Unit cost of procurement.
c_P	Unit cost of production.
c_L	Unit cost of lost sale.
$c_{avg,R}$	Average cost of repair per product with respect to different qualities (calculated in Phase 1).
$c_{avg,M}$	Average cost of disassembly per product with respect to different qualities (calculated in Phase 1).
f_C	Setup cost of procurement.
f_P	Setup cost of production.
f_R	Setup cost of repair.
f_M	Setup cost of disassembly.
$H_C(0)$	Initial stock level of the components inventory.
$H_S(0)$	Initial stock level of the final products inventory.

Continued on next page

Table 2 – *Continued from previous page*

$H_R(0)$	Initial stock level of the repair inventory.
$H_M(0)$	Initial stock level of the disassembly inventory.
$H_C(t)$	Stock level of the components inventory at period t.
$H_S(t)$	Stock level of the final products inventory at period t.
$H_R(t)$	Stock level of the repair inventory at period t.
$H_M(t)$	Stock level of the disassembly inventory at period t.
$S(t)$	Quantity of final products to be sent to the market at period t.

Table 3: Decision Variables in Phase 2

$CP(t)$	Number of components to be procured at period t.
$C(t)$	Number of components to be used in production at period t.
$R(t)$	Number of products from the repair inventory to be used in repair activity at period t.
$M(t)$	Number of products from the disassembly inventory to be used in disassembly activity at period t.
$\lambda_P(t)$	Zero-one variable to determine if production will occur at period t or not.
$\lambda_C(t)$	Zero-one variable to determine if procurement will occur at period t or not.
$\lambda_R(t)$	Zero-one variable to determine if repair will occur at period t or not.
$\lambda_M(t)$	Zero-one variable to determine if disassembly will occur at period t or not.

4.2.1. Fuzzy Programming Model

Model-1 represents a fuzzy mixed-integer programming model for optimization of the RL network under consideration. The objective function includes 5 parts: (I) holding costs for the four inventories in the RL network, including the repair, disassembly, final product and component inventories, (II) component procurement, production, repair and disassembly costs, (III) setup costs for respective activities, (IV) a lost sale cost and, (V) a disposal cost.

Constraint (1) is used to balance the repair inventory level at each period with the previous period. Repair inventory level at period t is calculated considering the repair inventory level at period $t-1$, number of products inspected in period t and sent for repair and number of products to be used for repair in period t. Since the number of inspected products to be sent for repair is uncertain, it is represented as a fuzzy number, and, consequently the constraint is fuzzy, too. Constraints (2) to (4) are similar to constraint (1), but for the disassembly, components and final product inventories. Constraint (5) restricts quantity of products to be sent to the market to be equal to or less than fuzzy demand. Additionally, constraints (6) to (9) are used to make sure that zero-one decision variables for procurement, production, repair and disassembly are set to one when there is any product being procured, produced, repaired or disassembled, respectively at each time period, where Y represents a large number. Furthermore, constraint (10) restricts λ decision variables to be either zero or one, while constraint (11) shows that all other variables are non-negative. Finally, constraint

(12) sets the quantity of procurement, production, repair and disassembly at time period 0 or before to be zero.

Model-1

$$
\begin{aligned}
&Minimise \sum_{t=1}^{T} [h_R\, H_R(t) + h_M\, H_M(t) + h_S\, H_S(t) + h_C H_C(t)] + && (I)\\
&\quad \sum_{t=1}^{T} [c_C CP(t) + c_P C(t)\ + c_{avg,R} R(t) + c_{avg,M} M(t)] + && (II)\\
&\quad \sum_{t=1}^{T} [f_C \lambda_C(t) + f_P \lambda_P(t)\ + f_R \lambda_R(t) + f_M \lambda_M(t)] + && (III)\\
&\quad \sum_{t=1}^{T} c_L(\tilde{D}(t) - S(t)) && (IV)\\
&\quad \sum_{t=1}^{T} c_G Defuzz(\widetilde{B_G}(t)) && (V)\\
&Subject\, to: && \\
&H_R(t) - H_R(t-1) + R(t) = \tilde{B}_R(t) \quad 1 \le t \le T && (1)\\
&H_M(t) - H_M(t-1) + M(t) = \tilde{B}_M(t) \quad 1 \le t \le T && (2)\\
&H_C(t) = H_C(t-1) + CP(t - LT_C) + M(t - LT_M) - C(t) \quad 1 \le t \le T && (3)\\
&H_S(t) = H_S(t-1) + C(t - LT_P) + R(t - LT_R) - S(t) \quad 1 \le t \le T && (4)\\
&S(t) \le \tilde{D}(t) \quad 1 \le t \le T && (5)\\
&Y\lambda_P(t) \ge C(t) \quad 1 \le t \le T && (6)\\
&Y\lambda_C(t) \ge CP(t) \quad 1 \le t \le T && (7)\\
&Y\lambda_R(t) \ge R(t) \quad 1 \le t \le T && (8)\\
&Y\lambda_M(t) \ge M(t) \quad 1 \le t \le T && (9)\\
&\lambda_P(t), \lambda_C(t), \lambda_R(t), \lambda_M(t) \in \{0, 1\} \quad 1 \le t \le T && (10)\\
&R(t), M(t), CP(t), C(t), H_R(t), H_M(t), H_C(t), H_S(t) \ge 0 \quad 1 \le t \le T && (11)\\
&C(t) = 0, CP(t) = 0, R(t) = 0, M(t) = 0 \quad t \le 0 && (12)
\end{aligned}
$$

$H_R(0), H_M(0), H_C(0)$ and $H_S(0)$ are inputs into the model.

4.2.2. Conversion to a crisp Integer Programming model

The fuzzy integer programming model, Model-1, needs to be converted into a crisp integer programming model to be solved using available solvers. Constraints (1-2) and (5) have fuzzy right hand sides. Part IV of the objective function includes a fuzzy term as well and, hence the objective function is also fuzzy. Different approaches to handling fuzzy mathematical programming problems with a fuzzy objective function, fuzzy coefficients and fuzzy constraints have been investigated and proposed (e.g. Inuiguchi and Ramik (2000); Herrera and Verdegay (1995); Cadenas and Verdegay (2006)). A modified approach based on the concept of symmetric fuzzy linear programming (Zimmermann, 2001) is proposed in this paper and described in Appendix B. It is applied to Model-1, generating Model-2 as follows:

Model-2

$$
\begin{aligned}
&Maximise \quad \alpha \\
&Subject\,to: \\
&\sum_{t=1}^{T}\left[h_R\,H_R(t)+h_M\,H_M(t)+h_S\,H_S(t)+h_C H_C(t)\right]+ \\
&\quad\sum_{t=1}^{T}\left[c_C CP(t)+c_P C(t)\;+c_{avg,R}R(t)+c_{avg,M}M(t)\right]+ \\
&\quad\sum_{t=1}^{T}\left[f_C\lambda_C(t)+f_P\lambda_P(t)\;+f_R\lambda_R(t)+f_M\lambda_M(t)\right]+ \qquad (0)\\
&\quad\sum_{t=1}^{T}c_L\left[\overline{D}(t)+(1-\alpha)\left(D_U(t)-\overline{D}(t)\right)-S(t)\right]+ \\
&\quad\sum_{t=1}^{T}c_G Defuzz(\widetilde{B_G}(t)) \\
&\quad\le f_{\min}+(1-\alpha)(f_{\max}-f_{\min})
\end{aligned}
$$

$$
\begin{array}{lrr}
H_R(t)-H_R(t-1)+R(t)\le \underline{B_R}(t)+(1-\alpha)\left(B_{R_L}(t)-\underline{B_R}(t)\right)+(1-\alpha)p_R & 1\le t\le T & (1)\\
H_R(t)-H_R(t-1)+R(t)\ge \overline{\overline{B_R}}(t)+(1-\alpha)\left(B_{R_U}(t)-\overline{\overline{B_R}}(t)\right)-(1-\alpha)p'_R & 1\le t\le T & (1')\\
H_M(t)-H_M(t-1)+M(t)\le \underline{B_M}(t)+(1-\alpha)\left(B_{M_L}(t)-\underline{B_M}(t)\right)+(1-\alpha)p_M & 1\le t\le T & (2)\\
H_M(t)-H_M(t-1)+M(t)\ge \overline{\overline{B_M}}(t)+(1-\alpha)\left(B_{M_U}(t)-\overline{\overline{B_M}}(t)\right)-(1-\alpha)p'_M & 1\le t\le T & (2')\\
H_C(t)=H_C(t-1)+CP(t-LT_C)+M(t-LT_M)-C(t) & 1\le t\le T & (3)\\
H_S(t)=H_S(t-1)+C(t-LT_P)+R(t-LT_R)-S(t) & 1\le t\le T & (4)\\
S(t)\le \underline{D}(t)+(1-\alpha)\left(D_L(t)-\underline{D}(t)\right)+(1-\alpha)p_D & 1\le t\le T & (5)\\
Y\lambda_P(t)\ge C(t) & 1\le t\le T & (6)\\
Y\lambda_C(t)\ge CP(t) & 1\le t\le T & (7)\\
Y\lambda_R(t)\ge R(t) & 1\le t\le T & (8)\\
Y\lambda_M(t)\ge M(t) & 1\le t\le T & (9)\\
\lambda_P(t),\lambda_C(t),\lambda_R(t),\lambda_M(t)\in\{0,1\} & 1\le t\le T & (10)\\
R(t),U(t),CP(t),C(t),H_R(t),H_M(t),H_C(t),H_S(t)\ge 0 & 1\le t\le T & (11)\\
C(t)=0,CP(t)=0,R(t)=0,M(t)=0 & t\le 0 & (12)
\end{array}
$$

where f_{min} and f_{max} represent approximations of the best and the worst network costs, respectively. p_R, p'_R, p_M and p'_M are tolerances for fuzzy quantities $\tilde{B}_R$ and $\tilde{B}_M$, respectively, and p_D is a tolerance for $\tilde{D}$.

5. Numerical Experiments

In order to gain an insight into the behaviour of the RL networks under consideration, different experiments are carried out. First, RL network performances with different remanufacturing and repair thresholds are obtained. The performance of each recovery policy, defined by the remanufacturing and repair thresholds, is calculated by determining outputs of the Phase 1 and inputting them into Phase 2, i.e. the fuzzy optimization model. Further on, the impact of various RL network parameters, including quantity of returned products, unit repair costs, unit production cost, setup costs and disposal cost on the RL cost incurred is analysed in the corresponding numerical experiments.

5.1. RL Network Parameters and Inputs

Parameters of a RL network used for the experiments are detailed in Table 4.

Table 4: Main RL network parameters

Activities		Unit Cost	Setup Cost	Lead Time
Production		30	1000	3
Repair	Quality level 1	160	1000	2
	Quality level 2	145		
	Quality level 3	120		
	Quality level 4	60		
	Quality level 5	10		
Disassembly	Quality Level 1	130	1000	4
	Quality level 2	105		
	Quality level 3	50		
	Quality level 4	20		
	Quality level 5	20		
Components Purchasing		100	1000	5
Lost Sale		150		

Inventories	Unit Holding Cost
Repair inventory	4
Disassembly inventory	3
Component inventory	5
Final product inventory	6

Please note that inspection cost is assumed to be negligible as it can be a trivial process. Also, disposal cost is initially assumed to be zero. For example, this is the case when the RL network is not responsible for the product disposal. Parameters relevant to the conversion of the fuzzy optimization model into a crisp model are tolerance values p_R, p'_R, p_M and p'_M which are set as 30% of the average quantity of returned products in the respective routes, $p_D = 4$, $f_{min} = 144500$ and $f_{max} = 255000$, where f_{min} is the best network cost determined approximately as the minimum cost which is incurred when recovery policy $P(4, 3)$ is applied and quantity of recovered products is equal to the demand and there is no uncertainty in both demand and returned products and f_{max} is the maximum network cost which is incurred when all the demand is lost with no returned products.

A time horizon of 25 unit time periods is considered. Fuzzy demand and return quantities of different quality levels for each period are presented in Table 5. It is assumed that there are five quality levels. It is worth noting that the total demand in the time horizon is represented by the trapezoidal membership function [1642,1679,1721,1758], while the total quantity of returned products is [1079,1178,1300,1399]. Defuzzified values of total demand

and returned products are 1700 and 1239, respectively. In addition, demand is zero for the first 8 periods to allow the RL network to prepare for supplying the demand (the lead time for the forward production route which is the longest lead time is 8).

Table 5: Fuzzy demand and fuzzy quantities of returned products with different quality levels

Time Period	Demand	Quantity of returned products of certain quality level				
		1	2	3	4	5
1	[0,0,0,0]	[9,10,12,13]	[10,11,11,12]	[8,8,10,10]	[8,8,10,10]	[8,9,9,10]
2	[0,0,0,0]	[8,9,9,10]	[7,7,9,9]	[9,11,11,13]	[9,10,10,11]	[10,11,11,12]
3	[0,0,0,0]	[8,10,10,12]	[8,9,9,10]	[9,10,12,13]	[9,9,9,9]	[8,8,10,10]
4	[0,0,0,0]	[9,9,11,11]	[10,10,12,12]	[8,10,10,12]	[7,8,8,9]	[8,9,9,10]
5	[0,0,0,0]	[7,8,8,9]	[11,11,11,11]	[8,8,10,10]	[9,10,10,11]	[10,11,11,12]
6	[0,0,0,0]	[9,10,10,11]	[8,10,10,12]	[8,10,10,12]	[9,9,11,11]	[8,8,10,10]
7	[0,0,0,0]	[8,9,11,12]	[8,9,11,12]	[9,10,12,13]	[7,7,9,9]	[10,10,12,12]
8	[0,0,0,0]	[8,8,10,10]	[9,10,10,11]	[7,9,9,11]	[8,9,11,12]	[7,9,9,11]
9	[95,98,102,105]	[12,13,13,14]	[8,8,10,10]	[9,10,12,13]	[8,8,10,10]	[9,11,11,13]
10	[96,100,100,104]	[8,9,9,10]	[9,10,10,11]	[9,10,12,13]	[9,10,10,11]	[8,8,10,10]
11	[95,99,101,105]	[8,9,11,12]	[10,10,12,12]	[9,10,10,11]	[8,10,10,12]	[9,9,11,11]
12	[95,98,102,105]	[10,11,13,14]	[8,8,8,8]	[9,10,12,13]	[8,9,9,10]	[9,9,9,9]
13	[98,99,101,102]	[9,9,9,9]	[8,8,10,10]	[10,11,11,12]	[8,9,9,10]	[8,10,10,12]
14	[96,98,102,104]	[9,9,11,11]	[9,11,11,13]	[8,9,11,12]	[10,10,12,12]	[9,10,12,13]
15	[98,98,102,102]	[8,9,11,12]	[9,10,12,13]	[8,9,9,10]	[9,10,12,13]	[7,8,8,9]
16	[99,100,100,101]	[8,9,9,10]	[7,8,10,11]	[8,9,11,12]	[7,8,10,11]	[10,10,10,10]
17	[98,98,102,102]	[9,10,12,13]	[9,10,10,11]	[11,12,12,13]	[9,9,11,11]	[7,9,9,11]
18	[97,99,101,103]	[8,8,10,10]	[9,9,11,11]	[8,9,11,12]	[11,12,12,13]	[9,10,12,13]
19	[96,98,102,104]	[8,9,9,10]	[8,9,9,10]	[9,9,11,11]	[7,9,9,11]	[8,9,9,10]
20	[97,99,101,103]	[10,11,11,12]	[9,9,11,11]	[9,11,11,13]	[9,10,10,11]	[9,10,10,11]
21	[99,100,100,101]	[7,7,9,9]	[11,11,11,11]	[9,9,11,11]	[9,10,10,11]	[9,9,9,9]
22	[95,99,101,105]	[10,10,10,10]	[8,9,11,12]	[8,10,10,12]	[9,9,11,11]	[7,8,10,11]
23	[95,99,101,105]	[10,11,11,12]	[8,9,9,10]	[9,10,12,13]	[10,10,12,12]	[8,9,9,10]
24	[97,99,101,103]	[7,7,9,9]	[9,10,12,13]	[9,11,11,13]	[8,9,11,12]	[8,9,9,10]
25	[96,98,102,104]	[8,9,11,12]	[11,12,12,13]	[9,9,11,11]	[8,8,10,10]	[10,10,12,12]

5.2. Recovery Policies

To better understand the behaviour of the RL network, the impact of different quality thresholds for repair and remanufacturing of returned products are examined. Quality thresholds have a great influence on the RL network performance as they influence the overall cost of recovery activities, inventories and the lost sale costs. However, these relationships are quite complex, because in addition to quality thresholds, other parameters also affect the network performance.

In the following experiments, 21 different recovery policies $P(QT_R, QT_M)$ are used, consisting of all possible combinations of repair and remanufacturing quality thresholds, where it is assumed that the quality is described as an integer in the interval [1..5]. Quality threshold for repair is always greater or equal to the disassembly threshold. In the case when they are equal, the returned products of that or higher quality are repaired, while the rest is disposed. Furthermore, when the repair quality threshold is 6, the repair route is not used at all, and, in the case when both thresholds are 6, neither repair nor remanufacturing routes are used and all returned products are disposed. The total quantity of repaired and remanufactured products and the average costs of these recovery activities per product are shown in Table 6. It is evident from the table that as the repair threshold increases from 1 to 6, the quantity of returned products sent to the repair route decreases and the average repair cost per unit time period decreases as well. The same applies when the disassembly threshold is increasing.

Table 6: Performance of the recovery routes with different recovery thresholds

Recovery policy	Total recovery cost	Average cost of repair per product ($c_{avg,R}$)	Total repair quantity	Crisp total repair quantity	Average cost of disassembly per product ($c_{avg,M}$)	Total disassembly quantity	Crisp total disassembly quantity
P(1,1)	123565	99.73	[1079,1178,1300,1399]	1239	—	[0,0,0,0]	0
P(2,1)	116186	84.80	[864,945,1041,1122]	993	130.00	[215,233,259,277]	246
P(2,2)	84206	84.80	[864,945,1041,1122]	993	—	[0,0,0,0]	0
P(3,1)	106191	64.55	[643,707,779,843]	743	117.40	[436,471,521,556]	496
P(3,2)	74211	64.55	[643,707,779,843]	743	105.00	[221,238,262,279]	250
P(3,3)	47961	64.55	[643,707,779,843]	743	—	[0,0,0,0]	0
P(4,1)	88132	35.05	[426,463,507,544]	485	94.34	[653,715,793,855]	754
P(4,2)	56151	35.05	[426,463,507,544]	485	77.07	[438,482,534,578]	508
P(4,3)	29899	35.05	[426,463,507,544]	485	50.00	[217,244,272,299]	258
P(4,4)	16999	35.05	[426,463,507,544]	485	—	[0,0,0,0]	0
P(5,1)	78411	10.00	[213,233,251,271]	242	76.22	[866,945,1049,1128]	997
P(5,2)	46429	10.00	[213,233,251,271]	242	58.60	[651,712,790,851]	751
P(5,3)	20180	10.00	[213,233,251,271]	242	35.45	[430,474,528,572]	501
P(5,4)	7280	10.00	[213,233,251,271]	242	20.00	[213,230,256,273]	243
P(5,5)	2420	10.00	[213,233,251,271]	242	—	[0,0,0,0]	0

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Table 6 – *Continued from previous page*

P(6,1)	80832	—	[0,0,0,0]	0	65.24	[1079,1178,1300,1399]	1239
P(6,2)	48846	—	[0,0,0,0]	0	49.19	[864,945,1041,1122]	993
P(6,3)	22602	—	[0,0,0,0]	0	30.42	[643,707,779,843]	743
P(6,4)	9700	—	[0,0,0,0]	0	20.00	[426,463,507,544]	485
P(6,5)	4840	—	[0,0,0,0]	0	20.00	[213,233,251,271]	242
P(6,6)	—	—	[0,0,0,0]	0	—	[0,0,0,0]	0

The RL network performance achieved under different recovery policies are reported in Table 7, including α (the satisfaction degree achieved in fuzzy optimization), the average cost, which is calculated as the total cost incurred in the network divided by total demand in the time horizon, percentage of demand satisfied through each route, including repair, remanufacturing, forward routes and lost sale, the total setup costs for all recovery and production activities, costs per unit for all recovery and production activities, holding costs of all inventories and lost sale costs.

Table 7: Performance of the RL network under different recovery policies

Recovery Policy	Satisfaction degree α	Average cost	% of supply				Total costs			
			Repair	Remanufacturing	Forward route	Lost sale	Setup	Recovery and production activity	Inventory holding	Lost sale
P(1,1)	0.36	126.43	72%	0%	26%	2%	12000	184176	14739	4009
P(2,1)	0.36	126.33	58%	5%	35%	2%	15000	177242	16600	5924
P(2,2)	0.42	122.50	57%	0%	35%	8%	11000	162199	13201	21857
P(3,1)	0.37	125.65	43%	14%	40%	2%	17000	175713	15211	5689
P(3,2)	0.42	122.71	43%	8%	47%	3%	17000	171735	12696	7178
P(3,3)	0.46	120.19	42%	0%	56%	2%	16000	173056	9389	5870
P(4,1)	0.40	123.80	28%	30%	40%	2%	19000	172152	14627	4685
P(4,2)	0.45	120.48	28%	21%	49%	2%	19000	166806	13236	5774
P(4,3)	0.48	119.12	27%	11%	60%	2%	17000	166069	13222	6218
P(4,4)	0.44	121.25	28%	0%	67%	5%	13000	168214	11346	13564
P(5,1)	0.38	125.21	14%	45%	39%	2%	20000	173737	14891	4224
P(5,2)	0.43	121.91	14%	33%	46%	6%	18000	158490	14262	16496
P(5,3)	0.45	120.99	14%	22%	60%	4%	17000	162583	15004	11099
P(5,4)	0.39	124.68	14%	11%	73%	2%	18000	177005	12550	4398
P(5,5)	0.31	129.73	15%	0%	84%	1%	15000	192581	10225	2741

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Table 7 – *Continued from previous page*

P(6,1)	0.29	130.91	0%	57%	40%	3%	16000	183444	14775	8335
P(6,2)	0.35	127.06	0%	46%	53%	1%	18000	182124	12255	3618
P(6,3)	0.36	126.32	0%	34%	62%	4%	18000	176290	10922	9524
P(6,4)	0.30	130.57	0%	23%	76%	1%	16000	191306	12205	2466
P(6,5)	0.19	137.95	0%	12%	88%	0%	16000	208110	10360	48
P(6,6)	0.08	144.57	0%	0%	100%	0%	12000	224132	9639	0

Table 7 shows that the $P(4,3)$ is the best policy for the network under consideration generating the lowest average cost.

In the next sections, the sensitivity of this outcome to different network parameter values is examined.

5.3. Quantity of returned products

Quantity of returned products has a considerable effect on the performance of RL networks. In extreme cases, a small return quantity can make recovery uneconomical because forward production is necessary, while a high return quantity may make forward route unnecessary. In order to understand the effect of quantity of return on the RL network, different quantities of return are tested and the results are presented in Figure 3 and Figure 4. The percentages refer to the percentage of demand that is returned, while average cost refers to the total RL network cost divided by total demand. Fuzzy quantities of returned products in each unit time period are generated in such a way as to make the total quantity of returned products equal to the corresponding percentage of total demand. It is worth noting that, in the main experiment, this value was roughly equal to 70%.

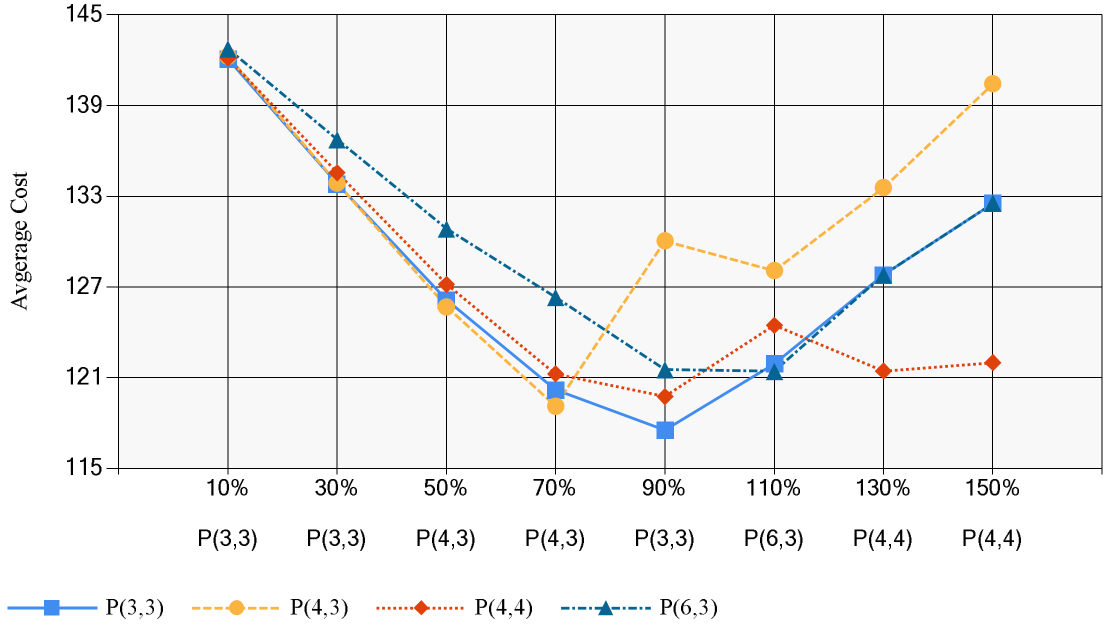

Figure 3: The average cost of the best policies for different return quantities

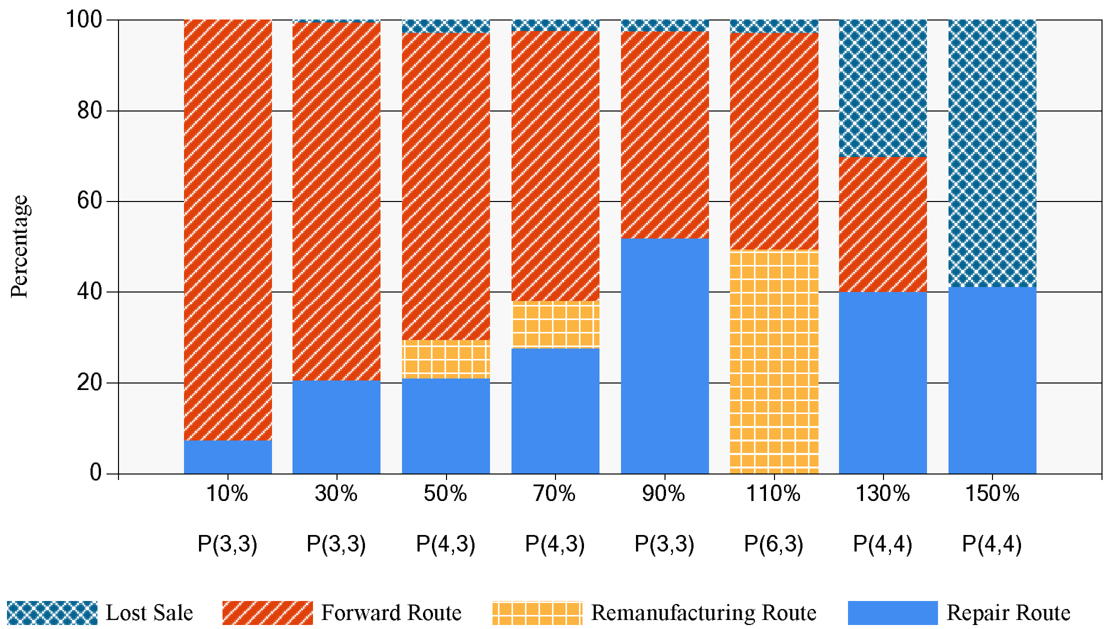

Figure 4: Percentage of the products supply of each route and the lost sale for different return quantities

Figure 3 shows the average cost of the RL network achieved for different quantities of returned products under different recovery policies. Percentages higher than 100 refer to cases when quantities of retuned products are higher than demand satisfied by the RL network under consideration (e.g., products are manufactured by other networks). The chart presents the best average cost incurred under different recovery policies and the corresponding best recovery policy. The best policies are $P(3,3)$, $P(4,3)$, $P(6,3)$ and $P(4,4)$. As it can be seen in Figure 3, policies perform similarly for lower return quantities, because demand is mainly satisfied from the forward route and not from the recovery routes and, hence, the average incurred costs under different recovery policies are similar. For higher return quantities, the differences in average cost are more noticeable. Also, one can see that three of the selected policies ($P(3,3)$, $P(4,4)$ and $P(6,3)$) perform the best at return quantity of 90% and the their incurred costs increase for both higher and lower than 90% return quantity. A trade-off between holding costs and setup costs is made. In the case of higher return quantities holding costs of disassembly and repair inventories are increased while less setup costs are incurred; for example, when the forward route can be completely avoided. For lower return quantities, less holding costs are incurred with higher setup costs.

Figure 4 represents the breakdown of different routes used to satisfy demand, expressed by percentages of demand satisfied through repair, remanufacturing and forward routes and the unsatisfied demand. The best recovery policy for each case of quantities of returns is printed below the x-axis labels. It can be concluded that for lower and higher quantities of returns only one recovery option is used as this leads to fewer setups and consequently lower setup costs. In the case of lower returns' quantities, demand is mainly satisfied from the forward route. In the case of higher returns' quantities, more demand is satisfied using the repair route than the forward route; this implies that returned products of good quality are used for repair and the rest is disposed. In the cases when the percentages of returns are between 50% and 70%, multiple recovery routes are used.

5.4. Unit Repair Costs

Repair costs have an effect on the RL network decisions on which routes to use for product supply. A change in unit repair costs could make the alternative recovery option (i.e. remanufacturing) more or less attractive for recovery of products of a particular quality level, but, also, it can affect the ways in which products are supplied, i.e. by forward production or recovery routes. In this experiment, various changes in unit repair costs are considered, expressed as percentages of the initial repair costs for different quality levels. The results are shown in Figure 5 and Figure 6. The best recovery policy is notified under the x-axis for each change in the unit repair costs.

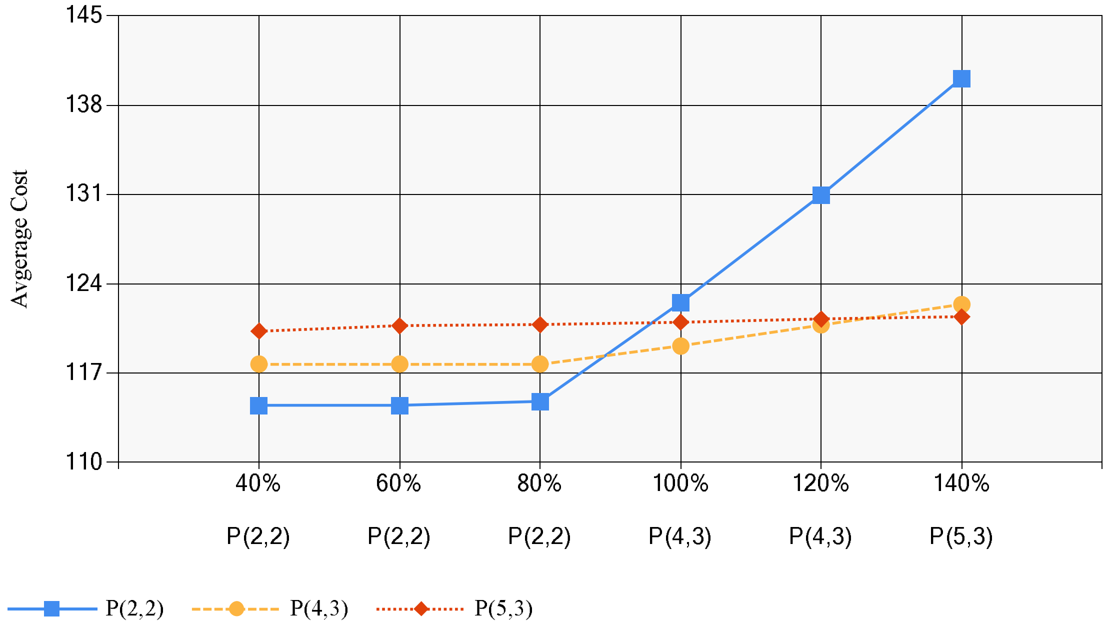

Figure 5: Comparison of the best policies average cost for different unit repair costs

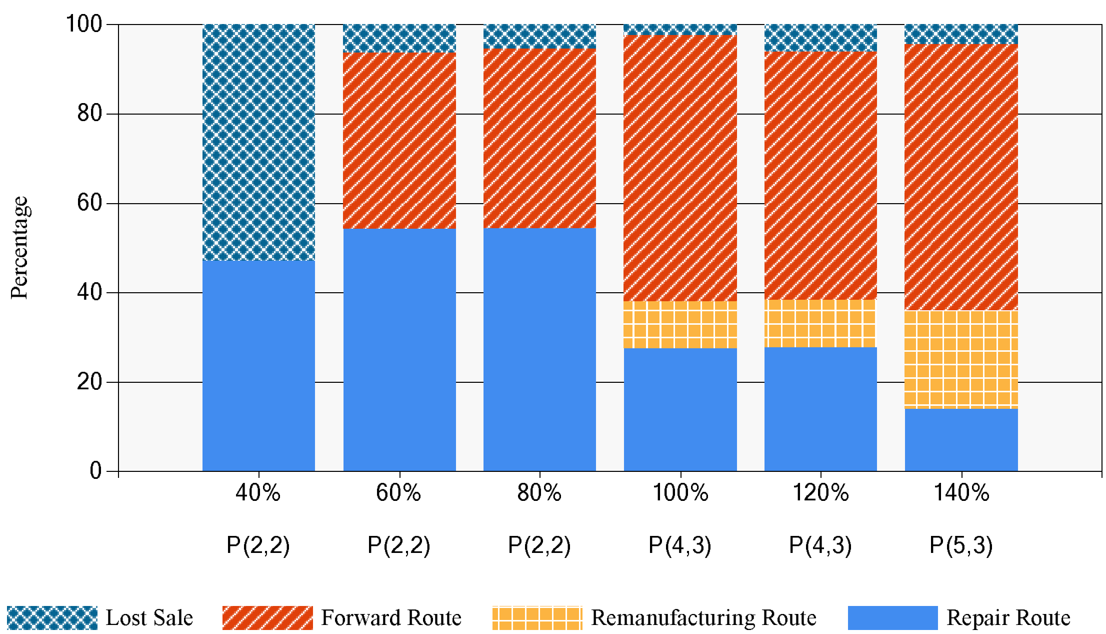

Figure 6: Percentage of the products' supply of each route and the lost sale for different unit repair costs

As one can see, in Figure 5, in the case of lower unit repair costs, the policy $P(2,2)$ which has a low quality threshold for repair and uses the repair route for recovery only, outperforms other policies. Furthermore, this policy is more sensitive to the increases in unit repair costs, and, as the repair costs increase, the average cost incurred increases rapidly. In contrast, average costs under recovery policies with higher repair thresholds (such as $P(4,3)$ and $P(5,3)$) are less sensitive to increases in the unit repair costs because less quantities of returned products are repaird. It is evident from Figure 6, that for lower unit repair costs, the repair route is used as well as the forward route, while for higher unit repair costs, the remanufacturing route as well as the forward route are used more as these are the cheaper alternatives.

5.5. Unit Production Cost

Production cost is incurred when the final product is produced either from a new component or from one recovered in the disassembly process. Therefore, unit production cost affects the operational costs of both remanufacturing and forward routes. In this experiment, sensitivity of the RL network to the unit production cost is considered where the unit production cost is changing from 40% to 160% of its original value ($c_p = 30$). The results are shown in Figure 7 and Figure 8.

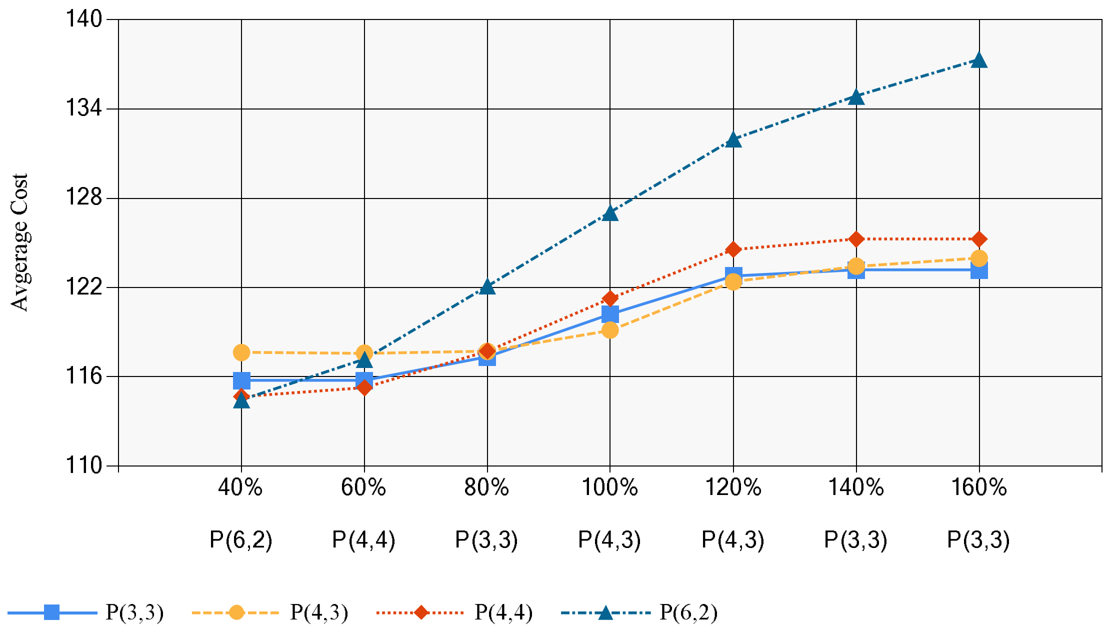

Figure 7: Average cost incurred under the recovery policies for different unit production costs

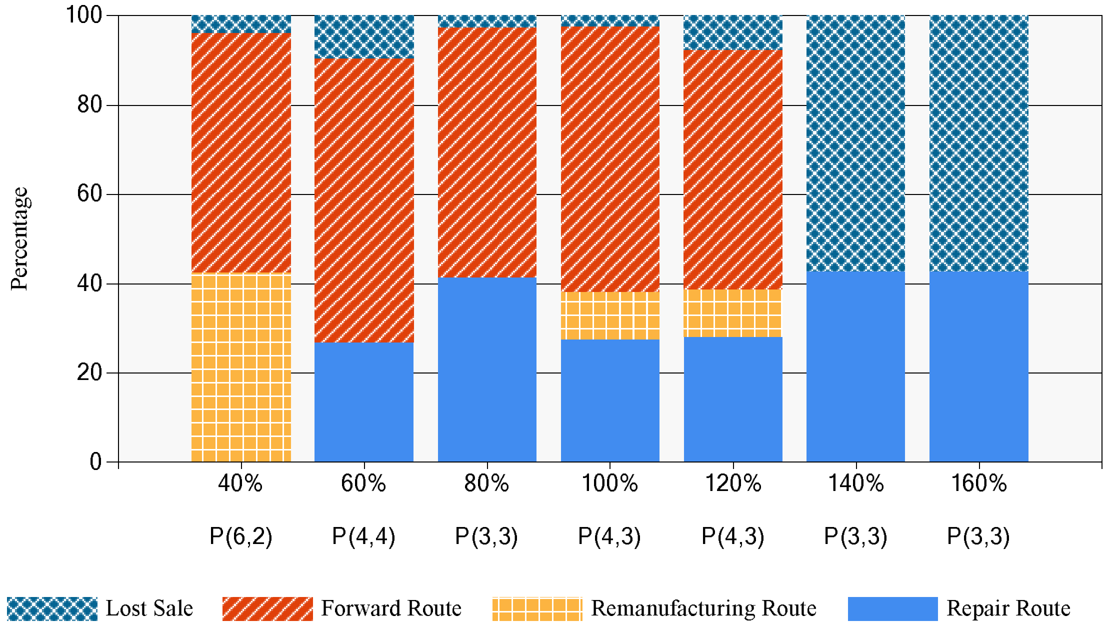

Figure 8: Percentage of the products supply of each route and the lost sale for different unit production costs

Figure 7 shows that policies which lead to higher remanufacturing quantity are generally more sensitive to the increases in unit production cost; for example, $P(6,2)$ compared to policy $P(4,3)$. This can be explained by the higher quantity of products which is supplied by remanufacturing route and, therefore, include the production activity. As it is evident in Figure 7 and Figure 8, policies which only use the remanufacturing route for recovery (such as $P(6,2)$) perform better for the lower unit production cost, while for the higher unit production cost, those policies which only use the repair route, while the rest of demand remains unsatisfied, incur lower average cost (for example, $P(3,3)$). Interestingly, in the middle of the range, the policies which utilise all three supply routes incur the lowest average costs (for example, $P(4,3)$).

5.6. Setup Costs

To understand the effect of setup costs on the average cost incurred in the RL network different values of setup costs are examined. All setup costs for repair, disassembly, production and component procurement are set to the same value which changes from 500 to 4000. The results are shown in Figure 9 and Figure 10.

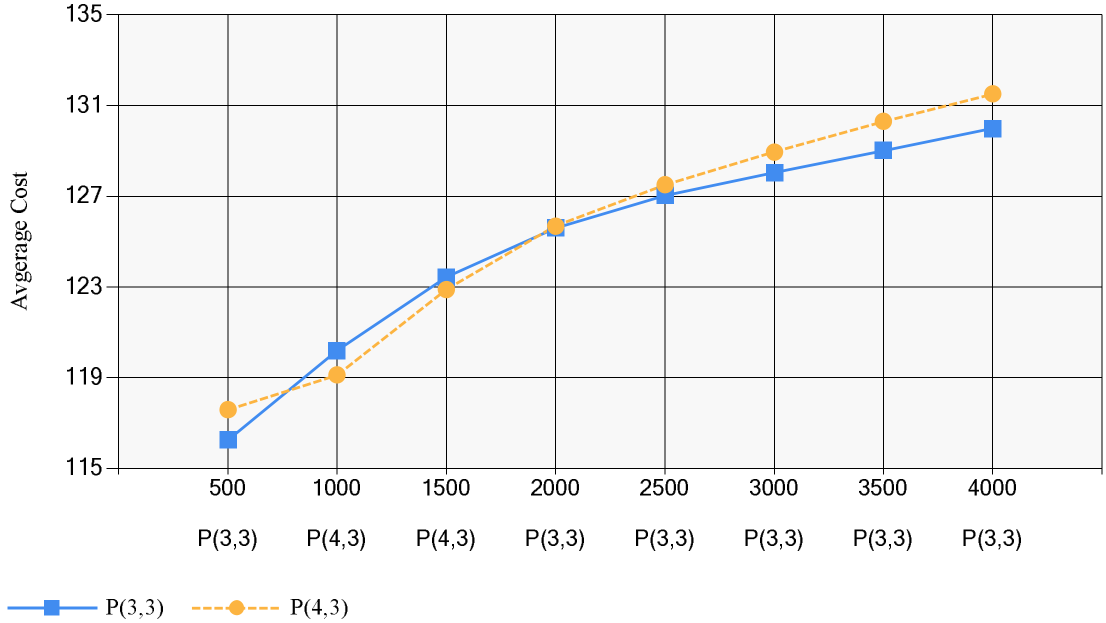

Figure 9: Average cost incurred under the recovery policies for different setup costs

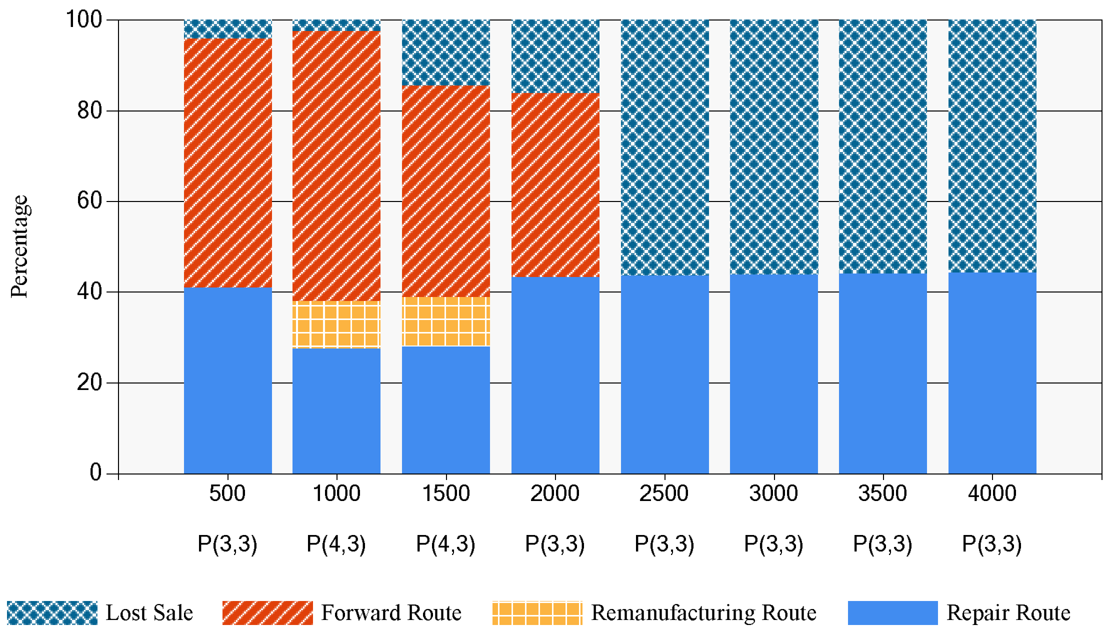

Figure 10: Percentage of the products supply of each route and the lost sale for different setup costs

It is evident from Figure 9 that as the setup costs increase, the average RL network cost also increases non-linearly. Also, one can see in Figure 10, in the case of a high setup costs it becomes economically inefficient to satisfy all demand, and, hence, there is an increase in the lost sale costs. Furthermore, having different batches in different recovery routes raises the setup costs, and, after some threshold in increasing the setup costs, it becomes more economical to operate one recovery route; therefore, for higher setup costs the repair only policy, $P(3,3)$, performs better than other policies.

5.7. Unit Disposal Cost

Unit disposal cost can have a significant impact on an RL network. High unit disposal cost can make recovery more attractive. On the other hand, low unit disposal cost can similarly make recovery less desirable. It is worth noting that negative unit disposal cost is also possible as the disposed products might be valuable and sold to another business. For example, tyres can be recycled into crumb rubber to be used in other products. However, such usage in the model is considered to be disposal of the product as the product leaves the RL network before being recovered.

In this experiment, different unit disposal costs are considered to better understand sensitivity of the RL network to this cost. Unit disposal costs in the range of -10 to 30 are examined. The results are presented in Figure 11 and Figure 12.

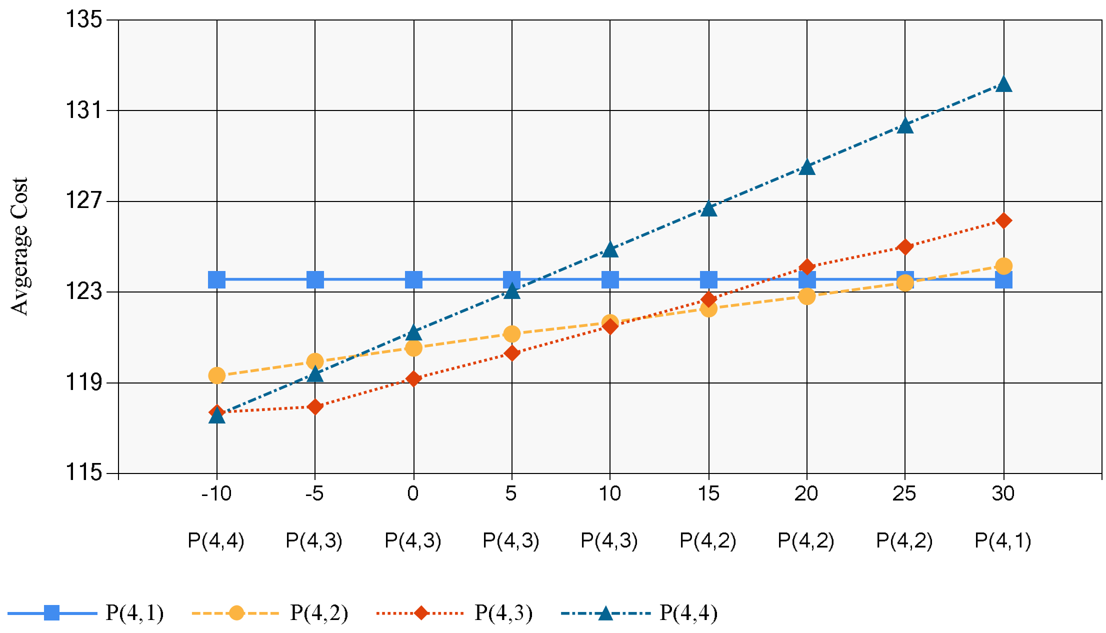

Figure 11: Average cost incurred under different recovery policies for different unit disposal costs

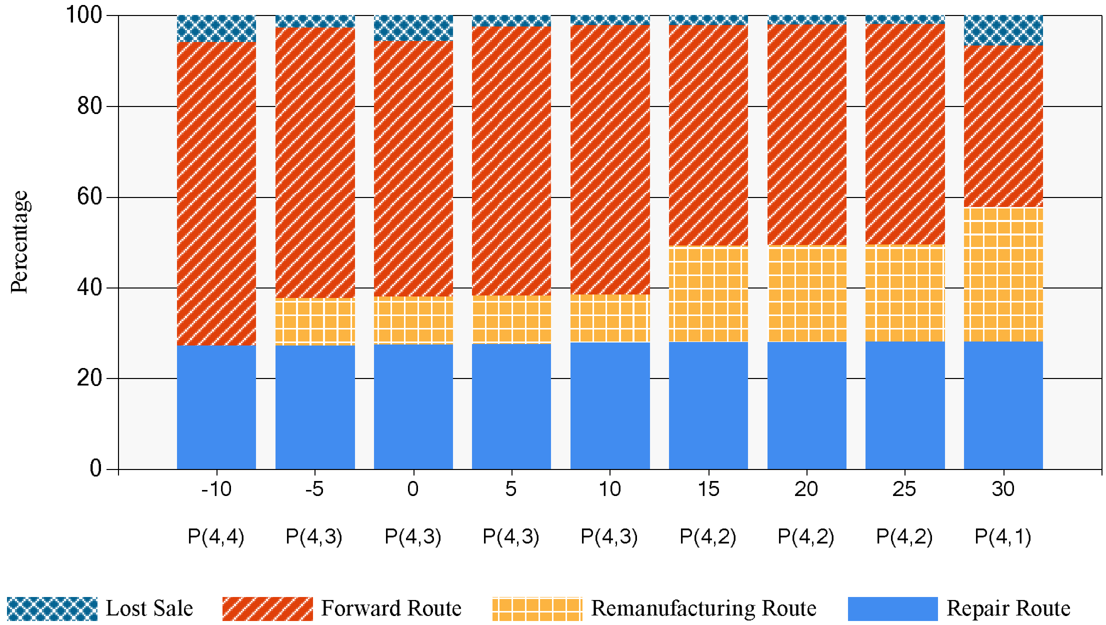

Figure 12: Percentage of the products supply of each route and the lost sale for different unit disposal costs

Figure 11 shows that an increase in disposal cost results in an increase in the average cost of all the recovery policies except when there is no disposal; such as the policy $P(4,1)$ for which the average cost remains constant. Evidently, in the case of the lower remanufacturing threshold, the RL cost is less affected by an increase in the unit disposal cost. Hence, the remanufacturing threshold is decreasing with increases in the unit disposal cost.

As it was expected and can be seen in Figure 12, the share of forward route was in an inverse proportion to the disposal cost. For unit disposal -10, the repair only policy $P(4,4)$ performs the best, while for unit disposal cost 30 the no disposal policy $P(4,1)$ was the best policy. Interestingly the unit disposal cost does not affect the quantity of repaired products. Possible reason is that the repair of the highest quality products is considered to be highly valuable and it is not affected by changes in the unit disposal cost.

6. Conclusions and Future Work

This paper is focused on RL networks with a forward route, two recovery routes, including repair and remanufacturing, and a disposal option. A fuzzy mixed integer programming model which facilitates decision making in presence of uncertainty in demand and quantity of returned products of different quality levels is developed. Quality of products is described as a scalar value which is utilised to separate the returned products into the two recovery and a disposal routes. Recovery policies are defined by two thresholds, namely repair and remanufacturing thresholds.

The RL network performances under different recovery policies are compared. It is concluded that recovery policies have a considerable impact on the RL network cost. Also, by carrying out numerical experiments, it is concluded that return quantity, unit repair costs, unit production cost, setup costs and disposal cost have impacts on the optimal recovery policy. Hence, a simple approach which assumes all returned products to be either recoverable or non-recoverable is not always realistic and can lead to inferior solutions.

The numerical experiments carried out have brought some insight into RL behaviour under different quality thresholds, i.e., recovery policies. The summary of the findings is as follows. First, the selection of recovery routes to be used in the networks have a great impact on the networks' performance. Further on, RL network parameters which have a considerable impact on selection of recovery policies are identified. It is shown that the repair route is used when there is a high quantity of returned products. The forward route is used for low quantities of return and the use of this route is decreasing while the quantity of returns is increasing. Repair only policies are cost effective for very high quantities of returns. It is proved that repair quality threshold decreases and increases with decreases and increases, respectively, in the unit repair cost. It seems that the remanufacturing quality threshold is not affected. In addition, for the lower unit production costs, the remanufacturing and forward production routes are more cost effective. For the higher unit production costs, the repair route becomes more economical, while remanufacturing and forward production routes are not used at all. With respect to the setup costs, mixed recovery policies, i.e. all recovery and forward production routes are used for certain range of the setup costs, while the repair route becomes dominant with increases in the setup costs. High unit disposal cost enforces the use of recovery routes, while the forward production route is used less, however, the unit disposal cost does not have an impact on the quantity of products to be repaired.

The future work will be carried out in the following directions. First, a model for determining quality thresholds for repair and remanufacturing routes in Phase 1 will be developed. Furthermore, this model will be enhanced to enable dynamic changes in the quality thresholds within the given time horizon. Second, the model will be extended to incorporate multi products and multi components. Finally, a multi objective optimization model which will consider not just the cost objective, but also environmental objectives will be developed and analysed.

Appendices

Appendix A. Fuzzy Arithmetic

Fuzzy trapezoidal numbers are used to represent uncertainty in both demand and return quantities of different quality levels. Fuzzy trapezoidal number $\tilde{a}$ is represented by a 4-tuple $(\underline{a}, a_L, a_U, \overline{a})$ with a membership function $\mu_{\tilde{a}}(x)$, as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x-\underline{a}}{a_L-\underline{a}} & \underline{a} \le x < a_L \\ 1 & a_L \le x \le a_U \\ \dfrac{\overline{a}-x}{\overline{a}-a_U} & a_U < x \le \overline{a} \\ 0 & otherwise \end{cases}$$

A trapezoidal membership function is graphically presented in Figure A.13.

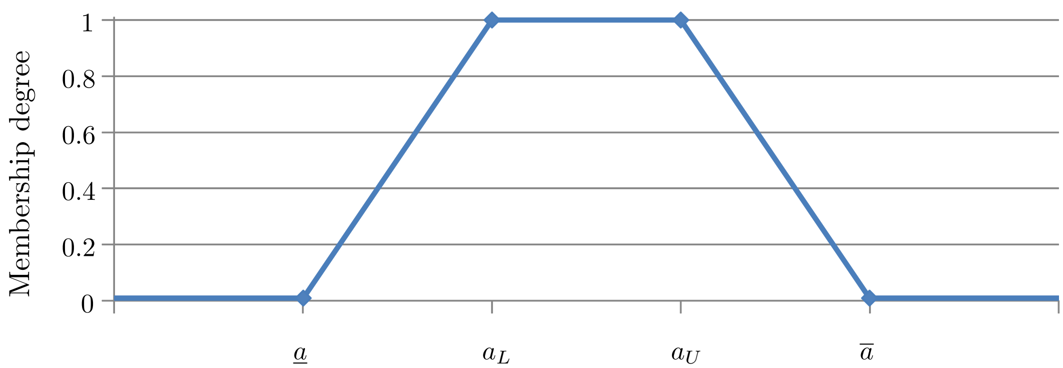

Figure A.13: Membership function of a trapezoidal fuzzy number $\tilde{a}$

Table A.8 shows fuzzy operators of trapezoidal fuzzy numbers used in this model, where $\tilde{a}$=$(\underline{a}, a_L, a_U, \overline{a})$ and $\tilde{b}$=$(\underline{b}, b_L, b_U, \overline{b})$.

Table A.8: Fuzzy operators

Operator	Syntax	Formula
Fuzzy Addition	$\tilde{a}+\tilde{b}$	$(\underline{a}+\underline{b}, a_L+b_L, a_U+b_U, \overline{a}+\overline{b})$
Fuzzy Subtraction	$\tilde{a}-\tilde{b}$	$(\underline{a}-\overline{b}, a_L-b_U, a_U-b_L, \overline{a}-\underline{b})$
Fuzzy Multiplication	$\tilde{a}*\tilde{b}$	$(\underline{a}*\underline{b}, a_L*b_L, a_U*b_U, \overline{a}*\overline{b})$
Fuzzy Division	$\tilde{a}/\tilde{b}$	$(\underline{a}/\overline{b}, a_L/b_U, a_U/b_L, \overline{a}/\underline{b})$
Scalar Multiplication	$m\tilde{a}$	$(m\underline{a}, ma_L, ma_U, m\overline{a})$
Defuzzification	$Defuzz(\tilde{a})$	$(\underline{a}+2a_L+2a_U+\overline{a})/6$

Appendix B. Conversion of fuzzy programming into crisp programming

A fuzzy linear programming model under consideration is as follows:

$$
\begin{aligned}
&\text{Find } X = [x_1, x_2, ..., x_n] \text{ which}\\
&minimise \quad \tilde{f}(X)\\
&s.t.\\
&\quad g_i(X) \leq \tilde{b}_i \qquad i = 1, 2, ..., m\\
&\quad g'_j(X) \geq \tilde{b}'_j \qquad j = 1, 2, ..., m'\\
&\quad 0 \leq X
\end{aligned}
$$

where $\tilde{f}(x)$ is a fuzzy objective function and $\tilde{b}_i$ and $\tilde{b}'_j$ are fuzzy parameters. They are modelled using trapezoidal membership functions as illustrated in Figure A.13. This optimization problem involves both fuzzy objective function and constraints expressed using fuzzy parameters. In the method proposed, they are both interpreted in the same way, by introducing satisfaction degree α in achieving the minimum of $\tilde{f}(x)$ and satisfying the constraints with fuzzy parameters $\tilde{b}_i$ and $\tilde{b}'_j$.

The fuzzy linear programming is converted into a crisp linear programming model as follows:

$$
\begin{aligned}
&maximise \quad \alpha\\
&s.t.\\
&\quad \overline{f}(X) + (1-\alpha)\left(f_U(X) - \overline{f}(X)\right) \leq f_{\min} + (1-\alpha)\left(f_{\max} - f_{\min}\right)\\
&\quad g_i(X) \leq \underline{b}_i + (1-\alpha)\left(b_{iL} - \underline{b}_i\right) + (1-\alpha)p_i \qquad i = 1, 2, ..., m\\
&\quad g'_j(X) \geq \overline{b}_j + (1-\alpha)\left(b_{jU} - \overline{b}_j\right) - (1-\alpha)p'_j \qquad j = 1, 2, ..., m'\\
&\quad 0 \leq X, \alpha \in [0, 1]
\end{aligned}
$$

In the crisp model, p_i and p'_j are tolerances introduced for fuzzy right hand sides which include $\tilde{b}_i$ and $\tilde{b}'_j$, respectively. Fuzzy tolerance values represent the maximum extent to which the constraint can be relaxed and represent the decision maker's intuition about the flexibility of parameters and constraints. In extreme cases, when the constraints cannot be relaxed at all $\alpha = 1$, while for $\alpha = 0$ constraints can be relaxed up to their tolerance values as follows:

$$
\begin{aligned}
&\text{when} \quad \alpha = 1:\\
&\overline{f}(X) \leq f_{\min}\\
&g_i(X) \leq \underline{b}_i \qquad i = 1, 2, ..., m\\
&g'_j(X) \geq \overline{b}'_j \qquad j = 1, 2, ..., m'
\end{aligned}
$$

$$
\begin{aligned}
&\text{when} \quad \alpha = 0:\\
&f_U(X) \leq f_{\max}\\
&g_i(X) \leq b_{iL} + p_i \qquad i = 1, 2, ..., m\\
&g'_j(X) \geq b'_{jU} - p'_j \qquad j = 1, 2, ..., m'
\end{aligned}
$$

In the case when a constraint with fuzzy parameters is expressed using equal relation, it is divided into two constraints, one less or equal constraint and one grater or equal constraint, and then converted into equivalent crisp constraints as described above.

The fuzzy objective function is transformed into a crisp constraint which limits the fuzzy values of the objective function with respect to the worst and the best possible objective function values, f_{max} and f_{min}, repsectively. In extreme cases, when the satisfaction degree reaches its maximum, $\alpha = 1$, the objective function should be lower than or equal to f_{min}, i.e. $\overline{f}(X) \leq f_{min}$; while for $\alpha = 0$, $f_U(X) \leq f_{max}$. Values f_{min} and f_{max} are determined depending on the problem under consideration.

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