A Fuzzy Dynamic Inoperability Input-Output Model for Strategic Risk Management in Global Production Networks

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Abstract

Strategic decision making in Global Production Networks (GPNs) is quite challenging, especially due to the unavailability of precise quantitative knowledge, variety of relevant risk factors that need to be considered and the interdependencies that can exist between multiple partners across the globe. In this paper, a risk evaluation method for GPNs based on a novel Fuzzy Dynamic Inoperability Input Output Model (Fuzzy DIIM) is proposed. A fuzzy multi-criteria approach is developed to determine interdependencies between nodes in a GPN using experts' knowledge. An efficient and accurate method based on fuzzy interval calculus in the Fuzzy DIIM is proposed. The risk evaluation method takes into account various risk scenarios relevant to the GPN and likelihoods of their occurrences. A

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case of beverage production from food industry is used to showcase the application of the proposed risk evaluation method. It is demonstrated how it can be used for strategic decision making by evaluating alternative GPNs.

Keywords

Risk Management, Inoperability Input Output Models, Global Production Networks, Supply Chain Management Risk, Fuzzy Arithmetic, Dynamic Models.

1 Introduction

A Global Production Network (GPN) is the set of globally interconnected actors (e.g. companies, groups, organisations, etc.) that enable the process of production or service provision from sourcing of the materials and labour to the consumption of the final products or provision of service (Coe et al., 2008). The actors within the GPN can include suppliers, production facilities, intermediaries, customers, service providers and consumer groups.

A few distinguishing features of GPNs which are of interest to strategic risk management are:
1) Global relationships; GPNs are facing unique challenges due to the involvement of trans-national actors and are especially affected by risks that can arise as a result of geo-political factors in a country or region. 2) Complexity of dependencies; interdependencies between actors are affected by various parameters related to GPN coordination among the actors and control procedures.
3) Complexity of networks; GPNs involves complex, often non-linear, relationships between the network partners, for example, they involve alternative suppliers for a particular material (Coe et al., 2008). All of these features can potentially contribute to the influence of risk on GPNs.

Risks in GPNs can be categorised, based on their cause, into different categories: (1) *internal* to the actors and are due to the specific characteristics of the actor such as reliability of equipment, safety inventory levels, quality issues, etc., (2) related to the *network* partners such as availability of raw material, suppliers' reliability, etc. from the supplier side, or risks related to customers, for example, uncertain demand, problem with order processing, payments, etc., and (3) *external* to the network, for example, various regulations, severe weather conditions or accidents, and geopolitical factors (Jüttner et al., 2003). We will consider all of these risk factors, in the context of GPNs at the strategic level and analyse interdependencies among risk factors and their propagation.

Decision making at the strategic level of GPNs is particularly challenging. While different information about potential actors and their regional characteristics are needed to make a strategic decision, such information is not necessarily available, at least not precisely. However, fuzzy set theory can be used to model the knowledge-related or epistemic uncertainties. Epistemic uncertainties refer to the reducible uncertainties that is caused by lack of knowledge about the subject, as opposed to aleatory uncertainties that are the irreducible uncertainties due to the random and stochastic nature of events (Kiureghian and Ditlevsen, 2009). In this paper, subjective knowledge, with inherent epistemic uncertainties is input by the experts in the form of linguistic labels such as Low demand in a certain region, Medium reliability of a certain supplier or High substitutability of a certain component and modelled as fuzzy parameters. Furthermore, fuzzy set theory is used to track epistemic uncertainties in the results obtained and to better understand the relationship between the uncertainties in the results and input parameters.

This paper is arranged as follows. Section 2 will introduce the relevant literature on risk management in GPNs, including propagation of risks, Inoperability Input Output models (IIM), Dynamic IIM (DIIM) and Fuzzy IIM. In Section 3, inoperability and risks in GPNs are considered and various aspects of the proposed method are discussed, including GPN configurations, regional and node specific risk factors, node interdependencies and their calculation using a multi criteria method, risk scenarios, discrete Fuzzy DIIM and economic loss of risk. Following this discussion, in Section 4, an example from the food industry is provided and used as an illustration of the proposed approach. Finally, the paper is concluded by discussing the outcomes and possible future directions in Section 5.

2 Literature Review

Risk management in GPNs is an important subject in the literature (Heckmann et al., 2015). We will focus on the propagation of risks in GPNs, and, in particular, the use of inoperability models to analyse propagation, dynamism in inoperability models, and, the application of fuzzy arithmetic to model uncertainty within the inoperability models.

2.1 Propagation of Risks in GPNs

A number of quantitative models have been proposed to observe and analyse consequences of risks and their dynamic propagation within a network, and, optimal decision making under risks. For this purpose, approaches such as simulation models, network theory and mathematical optimisation have been utilised.

Decision making under risks has been tackled using mathematical optimisation approaches. Gaonkar & Viswanadham (2007) investigated the propagation of events due to supplier non-performance and proposed two supplier selection models, one based on the Markowitz model for strategic level deviation management and the other based on a credit risk minimisation model which was introduced for disruptions management. Furthermore, Azaron et al. (2008) proposed a multi-objective stochastic programming approach for supply network design with the cost and risk objectives.

A bottleneck identification problem in supply networks is investigated by Mizgier, Jüttner & Wagner (2013) who utilised two methods based on network theory and Monte Carlo simulation, respectively. In the simulation model, systematic risks that affected all nodes of the supply network and the propagation of risk between the nodes are considered. Also, Taquechel (2010) applied network theory and fault trees to represent risk propagation and optimise the budget spending by minimising the risk in a US maritime supply network.

Bayesian Belief Networks (BBNs) are probabilistic models based on directed acyclic graphs where nodes have conditional dependencies that are represented using probabilities. Shin et al. (2012a) used BBNs to analyse supply network risks and both their interdependency and propagation throughout the supply network to determine a dynamic alternative path. Each node of the supply network was modelled by a BBN which represented interrelated risk indicators. All the BBNs influenced each other and in that way propagation of risk in the supply network was modelled. In a similar study, Shin et al. (2012b) considered risk based dynamic back order replenishment plans for multi echelon supply chains network stock-out and inventory costs. A heuristic method based on Reverse Dijkstra algorithm was proposed.

Simulation models are valuable tools to understand the behaviour of supply networks and they have been also used to model propagation of risk in the supply networks. For example, Bueno-Solano & Cedillo-Campos (2014) analysed the propagation of disruptions in supply neworks produced by terrorist acts using System Dynamics. Additionally, Sun, Xu & Hua (2012) applied agent-based simulation to bankruptcy propagation problems. Effects of a few contractual incentives, such as revenue sharing, price discount and quantity flexibility on bankruptcy propagation mitigation in multi manufacturer-multi retailer supply networks were examined. Also, Mizgier et al. (2015) investigated the diversification in global supply networks and analysed disruption propagation using Monte Carlo Simulation.

Furthermore, Wei, Dong & Sun (2010) utilised IIM to model propagation of risks in supply networks. The IIM considered the propagation effects of inoperability throughout the supply network and calculated the overall risk of inoperability of each node. Increasing the number of suppliers was suggested as a mitigation method and both the IIM and the mitigation method were validated using a Monte Carlo simulation model. In this paper, we will explore and propose a novel inoperability model.

2.2 Inoperability Models

The Input Output Model (Leontief, 1986) is a well-established economics model that is used to determine the relationship between interconnected sectors of economy. Each sector relies on products/services provided by other sectors, which creates interdependencies. Part of the necessary products/services is entered from outside, for example, from foreign markets, and it constitutes the input to the system. Also, part of the provided products/services is consumed by the final customers and/or exported, and this constitutes the output of the model.

The IIM is a risk model based on the Input Output model (Santos and Haimes, 2004). Similar to the Input Output Model, the IIM assumes interconnected nodes in a network. Risk is represented via independent “perturbations” caused by external events which have direct impact on some nodes in the network. A key concept behind the propagation of disruptions within networks is the interdependencies that exist between nodes. Interdependency between two nodes implies that the dependent node relies on the supporting node to function and, as a result, a disruption in the supporting node will affect the dependent node in proportion to the nodes’ interdependency. The model calculates inoperability values for all nodes by considering the propagation of perturbations throughout the network. Inoperability shows the rate at which the actual activity level at the node deviates from the planned activity level and acts as a measure of risk materialisation of each node.

The model is formulated in a vector format as follows:

$$\boldsymbol{q} = \boldsymbol{A}^{*}\boldsymbol{q} + \boldsymbol{c}^{*} \tag{1}$$

where $\boldsymbol{q}$ is the vector of nodes’ inoperabilities, $\boldsymbol{A}^{*}$ is the interdependency matrix, where each coefficient represents a degree of dependency and coupling of one node to the other, and $\boldsymbol{c}^{*}$ is the vector of input perturbations, modelled as normalised levels of disruptions which are directly induced by external events.

Santos & Haimes (2004) introduced a demand-reduction model that analysed the effects the inoperability of the sectors had on the demand of other sectors and also proposed a regional analysis within the IIM. Some of the important developments of IIMs include the consideration of inventories and their effect on the dynamic IIM (DIIM) (Barker and Santos, 2010) and agent-based IIM (Oliva et al., 2010). We will consider DIIM and Fuzzy IIM in more detail.

2.3 DIIM

DIIM extends the IIM by including time varying features of the network behaviour and operability. These models incorporate changes in perturbation values over time. A discrete-time DIIM which considers the resilience of a node to the change in inoperability caused by perturbation can be formulated in a vector format as follows (Haimes and Horowitz, 2005):

$$q(t+1) = KA^{*}q(t) + Kc^{*}(t) + (I-K)q(t) \tag{2}$$

where $q(t)$ is the inoperability vector of the nodes at time period t, K is the diagonal resilience matrix of nodes, A^{*} is the matrix of interdependencies between the nodes, $c^{*}(t)$ is the external perturbation of nodes at time period t and I is the identity matrix.

Haimes & Horowitz (2005) introduced DIIM, where time-varying effects were considered, using demand-reduction and regional models. A case study of HEMP attack was investigated using both BEA and RIMS-II datasets. Lian & Haimes (2006) expanded the DIIM, formulating a continuous version and included uncertainty in the form of stochastic – Brownian motion. Both the demand reduction and dynamic recovery scenarios were investigated based on a case study of terrorist attacks using the above mentioned datasets. Additionally, Baghersad and Zobel (2015) considered the product and service allocation preferences of different sectors and proposed a linear programming model.

The resilience factor is included in the dynamic IIM to represent the speed of individual node's response to changes in inoperability. For example, when recovering from a disruption, this factor shows the rate at which the node recovers. Its value is between 0 and 1, where 1 represents the fastest possible response and 0 represents no response at all.

Resilience depends on risk management practices applied at a node. The better the risk management implemented and the better its procedures, the higher the rate of recovery for the node.

The resilience factor k of a node can be determined by analysing the node's history of managing disruptions and the speed of recovery. The following formula can be used (Haimes and Horowitz, 2005):

$$k = \frac{-ln\left[\frac{q_s(T)}{q_s(0)}\right]}{T} \tag{3}$$

where k is a scalar specifying the resilience of the node, s represents a scenario where the node is recovering from a disruption, T is the number of periods that is needed for the node to reach 99% recovery from the disruption, $q_s(T)$ is the level of inoperability at time T and $q_s(0)$ is the initial level of inoperability caused by the disruption.

2.4 Fuzzy IIM

In the original inoperability models that are considering economic sectors, it is possible to determine the interdependencies based on statistical data that have been gathered nationally or regionally for the corresponding sectors. In a GPN, however, such information is not necessarily available. Especially at the early stages of GPN design, some of the partners can be new to the company and, hence, with no record to rely on for the statistical analysis. A manager often needs to make a strategic decision and relies on subjective judgement and expertise, which can be conveniently expressed using linguistic terms. Fuzzy numbers and arithmetic provide an appropriate framework for modelling these type of data and carrying out corresponding arithmetic operations in absence of empirical and historical data.

There are a few relevant papers on IIM that have used fuzzy sets to model uncertainty. Panzieri & Setola (2008) and Oliva, Panzieri, & Setola (2011) used triangular fuzzy numbers to represent interdependency and perturbation values in IIMs. Setola, De Porcellinis & Sforna (2009) used triangular fuzzy numbers with experts' reliability to assess interdependencies. Additionally, Oliva et al. (2014) examined the use of fuzzy difference inclusions for general discrete-time linear systems of the form $x(k+1) = H\,x(k)$.

In this paper, we will examine fuzzy time-varying perturbations in the DIIM with fuzzy resilience, which to the best of our knowledge has not been considered in the literature. A novel fuzzy DIIM that accommodates fuzziness in all the model parameters and uses interval calculations to determine inoperabilities in GPNs is introduced. A multi-criteria method is proposed for determining interdependencies among GPN nodes. The application of the fuzzy DIIM model is illustrated using a real world GPN in the food manufacturing sector which operates in the presence of risk. The impact of risks described in a risk scenario is measured using the fuzzy DIIM model proposed and applied in the evaluation of alternative GPNs.

3 Inoperability in GPNs

As discussed in the literature review, the concept of inoperability has been used frequently to represent the deviation of sectors of economy from their intended operation levels. In this paper, the same concept is adapted to GPNs, representing reduction in the activities of an individual network node in a GPN from its intended level. This can, for example, refer to a decrease in production levels from the expected values or reduction in demand in a customer segment. In this context, perturbation can be a direct disruption caused either by events that are external to the network, such as political or economic issues, or independent internal event at a network node, such as machine breakdown, insolvency, etc.

The GPN configuration is analysed to determine the interdependencies between network nodes and they are represented in the interdependency matrix. However, as statistical information is not necessarily available for these relationships, we rely on linguistic data which are collected from the experts and then are translated into fuzzy values. The interdependencies are determined using a multi criteria approach. We identified risks that can cause perturbation of the GPNs and defined risk scenarios which specify the timing, likelihood and impact of one or multiple risk factors that are likely to affect the network.

3.1 GPN Configuration

A GPN configuration is a particular arrangement of the company's production facilities with the relevant partners including suppliers, customers and service providers, with the target of providing a particular product or service. Various partners across the GPN can be represented as nodes such as supplier, producer, intermediary (e.g. logistics provider), customer, consumer or service provider. These nodes are connected with each other to form a network that represents the interdependency relationships between the nodes.

Multiple potential GPN configurations can be under consideration that can differ, for example, in the way suppliers, production facilities or customers are arranged or simply by the partner that is chosen to be the supplier. The main objective of the proposed risk analysis is to determine the relative suitability of potential GPN configurations with respect to risk and facilitate choosing the right GPN configuration.

3.2 Risk Factors

Disruptions can arise in the GPN due to many different risk factors. To analyse the risk in GPNs, it is necessary to identify and understand these factors and analyse their impact. We classify risk factors into two main groups: 1) regional risk factors caused externally to the GPN and 2) node specific risk factors.

3.2.1 Regional Risk Factors

To include global aspects of the GPNs, it is necessary to include region specific differences that exist around the globe. Distinct regions can be defined, depending on the desirable level of granularity, for example at city, country or continental levels. GPN nodes are assigned to the relevant region, and all risks that are relevant to a specific region can affect nodes within that region.

Regional risk factors can include a variety of risks including political, economic, social, technological, legal and environmental risks. These factors play an important aspect of GPNs as they account for differences in regions around the globe. In this paper, we consider two types of regional risk factors which can influence the nodes within the region:

1) Political instability due to war, political conflict, unrest, etc.
2) Economic issues caused by problems with currency fluctuations, inflation, etc.

3.2.2 Node Specific Risk Factors

Another type of risk factor can directly affect the nodes within GPN. This type of risk is a result of problems that are specific to the node and are analysed for each node separately. We analyse the following two examples:

1) Machine Malfunction where problems arise as a result of a defective machinery.
2) Insolvency where a financial situation of the node is in danger and insolvency and bankruptcy can arise.

3.3 Node Interdependencies

We propose a fuzzy multi-criteria method to estimate the interdependency among partners using experts' judgements. A list of nine dependency criteria is suggested to determine the interdependency between two nodes, dependent and supporting nodes:

1) Trade volume: the expected level of trade between two nodes. Interdependency has a direct relationship with trade volume; the higher the trade volume, the higher the inter dependency.
2) Inventory: the expected level of inventory kept between the nodes, either at the dependent node or the supporting node, a 3rd party or a combination of these. Interdependency has an inverse relationship with this criterion; the higher the stock, the lower the interdependency.
3) Substitutability of the product or service: considers if the product or service that is being delivered to the dependent node is replaceable. It can be replaced with a similar product that has higher availability in the market. There is an inverse relationship with interdependency; the higher the substitutability, the lower the interdependency.
4) Substitutability of the supplier/customer: if the supporting node, for example, a supplier or customer, can be replaced by another partner. The relationship is inverse; the higher the substitutability, the lower the interdependency.
5) Lead-time: the time it takes to receive an order from placing it. Interdependency has a direct relationship with lead-time; the higher the lead-time, the higher the interdependency, as it is most likely to need more time to react to any disruption.
6) Distance: the physical distance between the nodes. Interdependency has a direct relationship with the distance; the higher the distance, the higher the interdependency.
7) Information transparency: the amount of information that is being shared by the supporting node with the dependent node. Interdependency has a direct relationship with information transparency; the higher the information transparency, the lower the interdependency, as more information gives more chance to the dependent node to react to possible disruptions.
8) Collaboration agreement: considers how well the collaboration agreement is prepared and if it gives enough flexibility to the dependent node. Interdependency has a direct relationship with collaboration agreement; the more flexible the collaboration agreement, the lower the interdependency.

9) Compatibility of IT systems: considers if IT systems of the partners are compatible. Compatible IT systems allow for better and faster information sharing that improves responses to disruptions and hence, interdependency has an indirect relationship with compatibility of IT systems; the higher compatibility, the lower the interdependency.

The experts need to describe each link between two nodes considering two aspects: an estimated value of interdependency and the expert's confidence in the estimate. The estimated value can be described as either very low, low, fairly low, medium, fairly high, high or very high. The confidence is used to determine the corresponding uncertainty in the result obtained and can also be one of the mentioned linguistic labels.

The corresponding crisp values for the linguistic labels are assigned as shown in Table 1.

Table 1- Crisp values corresponding to the linguistic labels for both estimated value and confidence

Linguistic Label	Very Low	Low	Fairly Low	Medium	Fairly High	High	Very High
Crisp Value	0	0.167	0.333	0.5	0.667	0.833	1

Once the estimated value and the corresponding confidence are determined, a fuzzy interdependency weight for each link between the nodes and each interdependency criterion is expressed as a triangular membership function with a modal value equal to the corresponding crisp value. The triangular fuzzy number is defined in Appendix A. The left and right boundaries of the membership function are getting closer to the modal value, when confidence is increasing, and further away from the modal value, when confidence is decreasing.

The following formula for determining a triangular membership function for direct interdependency is used:

$$\widetilde{w_{r,l}} = \left(max\left(v_{r,l} - \left(1 - c_{r,l}\right), 0\right), v_{r,l}, min\left(v_{r,l} + \left(1 - c_{r,l}\right), 1\right)\right) \tag{4}$$

where $\widetilde{w_{r,l}}$ is the fuzzy interdependency weight of link l for criterion r, $v_{r,l} \in [0,1]$ is the crisp value corresponding to the estimated linguistic value of the link l for criterion r if link l is direct and $c_{r,l} \in [0,1]$ is the crisp value of the corresponding confidence value. For example, if confidence is very low, i.e. $c_{r,l} = 0$, then $\widetilde{w_{r,l}} = (0, v_{r,l}, 1)$, or when confidence is very high, i.e. $c_{r,l} = 1$, then $\widetilde{w_{r,l}} = (v_{r,l}, v_{r,l}, v_{r,l})$, i.e., it is a crisp number $v_{r,l}$.

The formula for inverse interdependency is formulated in a similar way. The modal value of the triangular membership function is calculated as $1 - v_{r,l}$, while the left and right boundaries are again getting closer to (further away from) the modal value when the confidence is increasing (decreasing).

To aggregate the interdependencies based on the fuzzy weights of all criteria, we use an Ordered Weighted Averaging (OWA) method, in line with Wei, Dong, & Sun (2010). This method aggregates the fuzzy weights, giving more importance to the criteria with higher weights. The advantage of using the OWA method is that criteria with higher weights, which suggest higher interdependency of a link with respect to these criteria, will have a higher effect than the criteria that have lower weights. For example, if a link is considered to have high dependency due to a low substitutability of the supplier and a high trade volume, but it is considered less dependent due to other criteria such as lead-time and distance, it will still be considered as a high interdependency link, as the two criteria with high weights will be considered more important than the ones with lower weights.

The following formula for the OWA aggregation is proposed:

$$\widetilde{w_l^*} = \left(\sum_{r=1}^{R} y_{r,l} \, \widetilde{w_{r,l}} \right) \Big/ L \qquad (5)$$

where $\widetilde{w_l^*}$ is fuzzy interdependency of link l on all criteria relative to number L of links to the node, R is the total number of criteria that has been rated, $y_{r,l}$ is the importance assigned to the criterion r, and, L is the total number of dependency links of dependent node i. The calculation requires summation and scalar multiplication of fuzzy numbers described in Appendix A.

The following formula for determining the criteria importance $y_{r,l}$ is proposed:

$$y_{r,l} = \frac{2(R - f(r) + 1)}{R(R + 1)}; \mathrm{r} = 1, \dots, \mathrm{R} \tag{6}$$

where $f(r)$ gives the position of criterion r in the sorted vector of interdependency criteria weights. It is determined by sorting the criteria in a descending order based on the modal of their fuzzy weights $\widetilde{w_{r,l}}$, and, then ranking them consecutively from 1 to R. For example, if the total number of criteria of link l is R = 3 and the modals of interdependencies of link , $\widetilde{w_{1,l}}$, $\widetilde{w_{2,l}}$ and $\widetilde{w_{3,l}}$ are 0.5, 0.1 and 0.2, respectively, then $f(1)$, $f(2)$ and $f(3)$ are 1, 3 and 2, respectively. As a result, criteria importances $y_{1,l}$, $y_{2,l}$ and $y_{3,l}$ will become $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$, respectively.

Assuming that empirical data is available for any of the criteria, the fuzzy number of interdependency can be conveniently replaced with a normalised value of the collected data. The normalisation should be carried out with regard to the minimum and maximum values that are historically established for the criterion. For example, in the case of trade volume, one needs to look at the historical information about the dependents relationships with the same or similar suppliers/customers and determine the minimum and maximum trade volumes. The normalisation method to be used depends on whether the relationship with the interdependency is direct or inverse. Higher normalised value, as with the fuzzy rating approach, should always be associated with higher interdependency.

3.4 Risk Scenario

To evaluate different GPN configurations with respect to their susceptibility to various risks, different risk scenarios need to be considered. A definition of a risk scenario includes a fixed time horizon for analysis, a sequence of the types of risk that causes perturbations of a node in the GPN of the considered configurations, level of perturbations, starting perturbations' time and end time. These scenarios can be defined based on historical data and/or experts' judgements.

Perturbation level is represented by a triangular fuzzy number in the range of 0 to 1, 0 representing no perturbation and 1 representing a total disruption of the node activities. Each scenario is assigned a likelihood of occurrence.

Causal links between risk factors are explicitly considered in a risk scenario. For example, knowing that the political issues are causing economic risks, implies that a risk scenario will define two perturbations, possibly with different levels and the corresponding starting and ending times.

The experts are expected to describe the likelihood of a scenario and perturbation level by considering two aspects: the estimated value and the expert's confidence in the estimate. Both are either specified directly as triangular fuzzy numbers or by using the linguistic labels, described in Table 1, and combined to form a fuzzy number using Formula (4).

In this paper, we assume that the estimates are based on the estimates of a single expert. However, it is possible to extend the method to multiple experts by considering a weighted average of their opinion as proposed by Setola et al. (2009).

3.5 Discrete Fuzzy DIIM

The following discrete fuzzy DIIM is proposed:

$$\tilde{q}(t+1) = \tilde{K}\widetilde{A^*}\tilde{q}(t) + \tilde{K}\widetilde{c^*}(t) + \left(I - \tilde{K}\right)\tilde{q}(t) \tag{7}$$

where $\tilde{q}(t)$ is the vector of fuzzy inoperability values of the nodes at time period t, $\tilde{K}$ is the fuzzy diagonal resilience matrix of nodes, $\widetilde{A^*}$ is the matrix of fuzzy interdependencies between the nodes, $\widetilde{c^*}(t)$ is the fuzzy external perturbation of nodes at time period t and I is the identity matrix. In this paper, it is assumed that all fuzzy parameters are modelled using triangular fuzzy numbers, although the proposed algorithm can work on any LR fuzzy number (Pedrycz and Gomide, 1998). Triangular fuzzy numbers are often used in applications due to their easy interpretability; they can conveniently represent standard linguistic terms such as “about a certain value” or “close to a certain value”. Also, generally calculations on triangular fuzzy numbers are easy to carry out.

In order to determine fuzzy inoperability values in Equation (7), a novel method based on fuzzy extension principle given in Appendix A and interval arithmetic is developed. An advantage of this method is that all parameters, including perturbations, interdependencies and resilience, can be fuzzy. The developed method provides an accurate and efficient way to carry out fuzzy arithmetic in DIIM, instead of using approximations. The foundation of this method and calculation procedure is described in detail in Appendix B.

It is worth mentioning that the definition of a period is flexible and can vary between different companies. It could potentially be between a day to a few months. However, in the example provided, we will assume that a period represents a week.

3.6 Economic Loss of Risk

The proposed fuzzy DIIM provides useful information about the operability level of GPN’s nodes that can be used to asses a particular GPN configuration. However, to assess suitability of a GPN, both perspectives, inoperability and economic benefit, need to be considered simultaneously. For example, a GPN configuration with a very low inoperability that has no economic benefit is not suitable, as well as a configuration that has a high economic benefit but with very high inoperability. Hence, the concept of economic loss of risk is introduced to allow for estimating the economic effect of risk.

Economic loss of risk for a node in a GPN at a certain time is calculated as the product of the intended revenue, that can be achieved by the node when fully operable during one time period, and the inoperability of the node at the time (Wei et al., 2010). The intended revenue can be, for example, the intended value of the products produced by a manufacturer, the value of supplies provided or the value of the products bought by the customer at a certain period of time.

To assess a GPN configuration from the risk perspective, the following formula is used to calculate the total economic loss of risk for a particular risk scenario:

$$Q_s = x' \sum_{t=1}^{T} q_s(t) \tag{8}$$

where Q_s is the total economic loss of risk for the GPN configuration in risk scenario s, T is the number of time periods in the time horizon under consideration, x' is the transpose of vector of intended revenue of all nodes for a single time period, $q_s(t)$ is the inoperability vector of all nodes at time period t for risk scenario s.

This can be further aggregated for all scenarios as follows:

$$Q = \sum_{s=1}^{S} p_s Q_s \tag{9}$$

where Q is the expected total economic loss of risk for the GPN configuration in all risk scenarios, S is the number of risk scenarios and p_s is the likelihood of risk scenario s.

4 An Example from Food Industry

The proposed risk model has been developed and implemented within FLEXINET – Intelligent Systems Configuration Services for Flexible Dynamic Global Production Network. It includes three industrial collaborators from different sectors. The process of adapting the model and its application in their practices is in progress. One of the collaborators is in the beverage sector. Based on this, we defined an illustrative example of a GPN of a drink manufacturing company that includes suppliers of fruits, certain ingredients and bottling products. It has two production facilities, one for fermentation and another for bottling. Most of company's operation is concentrated in Region 1, while the supplier of ingredients and one of the possible suppliers of bottling products are in another region (Region 2). The company is facing a choice between two suppliers of bottling products, one residing in Region 1 and the other in Region 2. The diagram representing the flow of material within this GPN is shown in Figure 1.

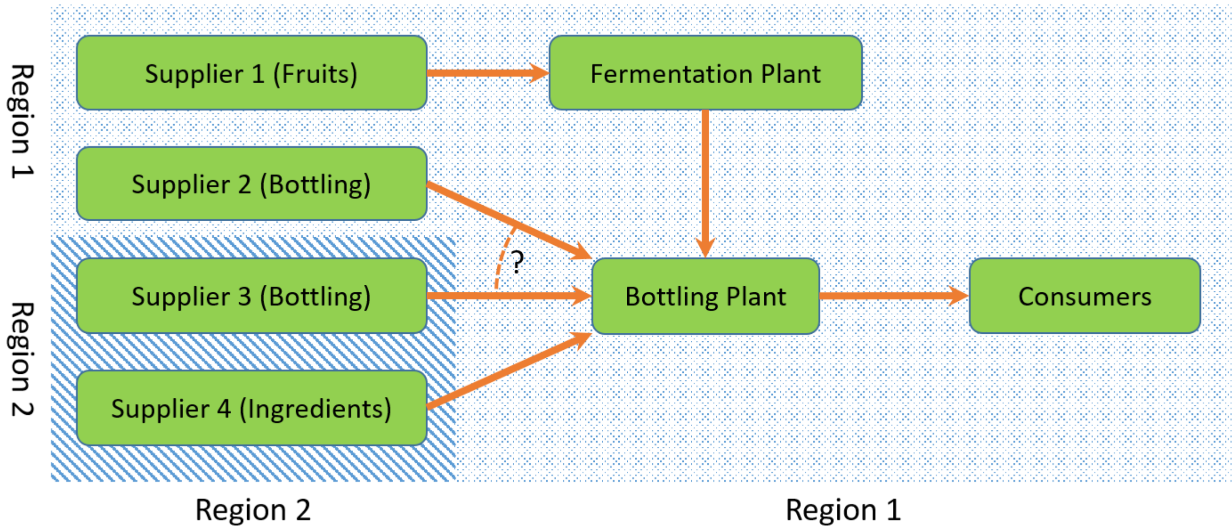

Figure 1- Configuration of a GPN in food industry

Relationships in the GPN lead to interdependencies between its nodes. It is worth mentioning that interdependencies exist not just between the links of the flow of material; other types of dependencies can exist in the reverse links. For example, while Fermentation Plant is dependent on Supplier 1 for the supply of fruits, Supplier 1 can also be dependent on Fermentation Plant as it may be the main customer of a particular type of fruits in the region. However, we will assume that Supplier 4 is not dependent on Fermentation Plant, as it has many alternative customers.

We consider two network configurations. Configuration 1 uses Supplier 3 (in Region 2) for bottling products, while Configuration 2 utilizes Supplier 2 (in Region 1). These two configurations will be compared in terms of their inoperability and economic loss of risk. Diagrams of the GPN

Configurations 1 and 2 and the interdependency links between the GPN nodes are presented in Figure 2.

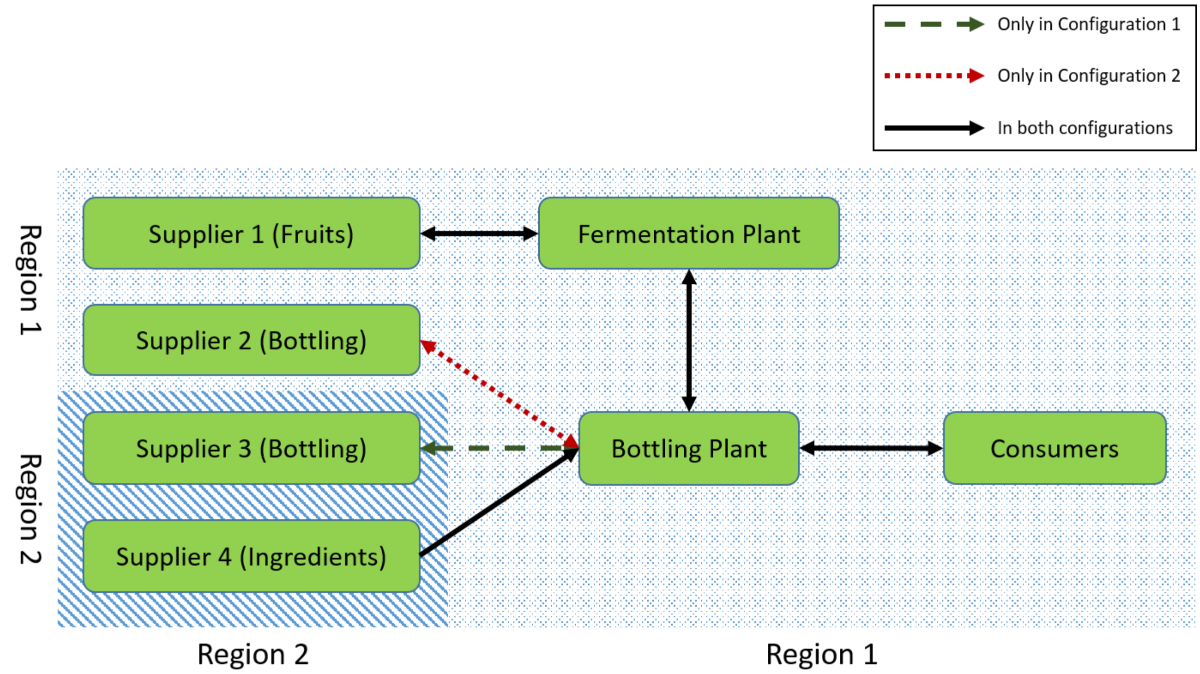

Figure 2- Interdependencies between the nodes in Configuration 1 and Configuration 2

4.1 Calculating Interdependencies

As described in Section 3.3., interdependencies are calculated using the fuzzy multi criteria method. For example, the interdependency value of Fermentation Plant on the Supplier 1 is calculated based on criteria values and the corresponding confidence presented in Table 2 and using Equation (4).

Table 2- Linguistic labels of criteria of interdependency of Fermentation Plant on Supplier 1

Criteria	*Value*	*Confidence*
Trade volume	Fairly High	High
Inventory	Fairly High	Medium
Substitutability of the product	Low	High
Substitutability of the supplier/customer	High	High
Lead-time	Medium	Medium
Distance	Very Low	High
Information transparency	Medium	High
Collaboration agreement	Low	Very High
Compatibility of IT systems	Low	Very High

Using the judgements provided in Table 2, the fuzzy dependency value of Fermentation Plant on Supplier 1 is calculated using Equations (5) and (6) as (0.507, 0.663, 0.833) / 2 = (0.254, 0.332, 0.417). Please note that the obtained value is divided by the number of dependencies of Fermentation Plant which is 2, including Supplier 1 and Bottling Plant. Using the same method, all fuzzy interdependency values shown in Figure 2 are calculated for both Configurations 1 and 2 and presented in Table 3.

Table 3- Interdependency values for both GPN configurations

From Node	**To Node**	**Fuzzy Interdependency Value**	
		Configuration 1	**Configuration 2**
Supplier 1	Fermentation Plant	(0.254, 0.332, 0.417)	(0.254, 0.332, 0.417)
Fermentation Plant	Supplier 1	(0.704, 0.759, 0.815)	(0.704, 0.759, 0.815)

From Node	To Node	Fuzzy Interdependency Value	
		Configuration 1	Configuration 2
Supplier 2	Bottling Plant	---	(0.102, 0.135, 0.169)
Bottling Plant	Supplier 2	---	(0.199, 0.389, 0.593)
Supplier 3	Bottling Plant	(0.139, 0.168, 0.196)	---
Bottling Plant	Supplier 3	(0.310, 0.565, 0.819)	---
Supplier 4	Bottling Plant	(0.110, 0.135, 0.160)	(0.110, 0.135, 0.160)
Fermentation Plant	Bottling Plant	(0.123, 0.136, 0.149)	(0.123, 0.136, 0.149)
Bottling Plant	Fermentation Plant	(0.185, 0.227, 0.269)	(0.185, 0.227, 0.269)
Bottling Plant	Consumers	(0.635, 0.690, 0.770)	(0.635, 0.690, 0.770)
Consumers	Bottling Plant	(0.178, 0.178, 0.178)	(0.178, 0.178, 0.178)

The values of interdependencies between Bottling Plant and Supplier 2 in Configuration 2 are lower than the interdependencies between Bottling Plant and Supplier 3 in Configuration 1. This is due to the fact that there is a smaller distance and lead-time, better collaboration agreement, information transparency and compatibility of IT systems between Supplier 2 and the Bottling Plant than that of the Supplier 3 and Bottling Plant.

Fuzzy resilience, that represents the recovery performance of a node, and the fuzzy intended revenue, which determines the economic loss that is incurred for each unit of inoperability (i.e. complete inoperability for one time period in the node), are set as shown in Table 4.

Table 4- Fuzzy resilience and intended revenue values for all nodes in the network

Node	Resilience	Intended Revenue

Node	Resilience	Intended Revenue
Supplier 1	(0.8, 0.9, 1)	(0, 0, 0)
Supplier 2	(0.8, 0.9, 1)	(0, 0, 0)
Supplier 3	(0.7, 0.8, 0.9)	(0, 0, 0)
Supplier 4	(0.7, 0.8, 0.9)	(0, 0, 0)
Fermentation Plant	(0.8, 0.9, 1)	(1500, 2000, 2500)
Bottling Plant	(0.8, 0.9, 1)	(2500, 3000, 3500)
Consumers	(0.5, 0.6, 0.7)	(0, 0, 0)

As evident from Table 4, the resilience of nodes in Region 1 is generally considered to be higher than in Region 2, except of Consumers which have lower resilience, i.e. they will be slower to respond and manage disruptions. The loss of risk is zero for all suppliers and consumers, as their loss will not directly affect the company. However, there are intended revenues defined for both of the company's production facilities, i.e. Fermentation Plant and Bottling Plant.

4.2 Risk Scenarios

Based on risk factors identified above, four relevant risk scenarios that can affect the network are defined. Scenarios 1 to 3 include two regional risks, both regional and node specific risks, a node specific risk only, respectively, while Scenario 4 considers a node specific risk associated with Supplier 2, which is relevant to Configuration 2, but not Configuration 1. A time horizon of 50 time periods is analysed. We are assuming that a period represents a week. They are shown in Table 5.

Table 5- List of risk scenarios used for the network evaluation

Scenario	Likelihood	Risk Factor	Affected	Perturbation	Period
Scenario 1	(0.1, 0.15, 0.2)	Political Instability	Region 2	(0.1, 0.2, 0.3)	1 to 10

		Economic Issues	Region 2	(0.05, 0.1, 0.15)	5 to 14
Scenario 2	(0.05, 0.1, 0.2)	Economic Issues	Region 1	(0.1, 0.2, 0.3)	1 to 10
		Insolvency	Supplier 1	(0.2, 0.3, 0.4)	1 to 15
Scenario 3	(0.3, 0.4, 0.5)	Machine Malfunction	Bottling Plant	(0.8, 0.9, 1)	1
Scenario 4	(0.1, 0.2, 0.3)	Insolvency	Supplier 2	(0.1, 0.2, 0.3)	1 to 10

4.3 Analysis of Results

An inoperability timeline shows the changes in the inoperability value of a particular node over the time horizon. Since the inoperability value is fuzzy, it is represented by different α-cuts' lower and upper endpoints, Min and Max, respectively. The α-cut of a fuzzy number is defined in Appendix A. The lines obtained for $\alpha = 0, 0.25, 0.5, 0.75$ and 1 using the discrete Fuzzy DIIM presented in Section 3.5 are shown in Figure 3 to Figure 7. An example of the inoperability timeline of the Fermentation Plant in Scenario 2 of Configuration 1 is shown in Figure 3 where y-axis is the level of inoperability and the x-axis is the timeline. For example, for $\alpha = 0$, there are two lines, one for the lower endpoint that is the most optimistic value for the inoperability of the node and another for the upper endpoint that is the most pessimistic value for the inoperability. The distances between these two lines correspond to uncertainty in inoperability; the larger the distance the more uncertainty in inoperability. For $\alpha = 1$, there is only one line which models the most likely values of the inoperability over the time horizon, i.e., its modals, in this case, the upper and the lower endpoints are the same. It might be interesting to notice that the upper and lower endpoints are not necessarily symmetrical to the point, modal, with the most likely value; this point can be nearer to the most pessimistic or most optimistic inoperability.

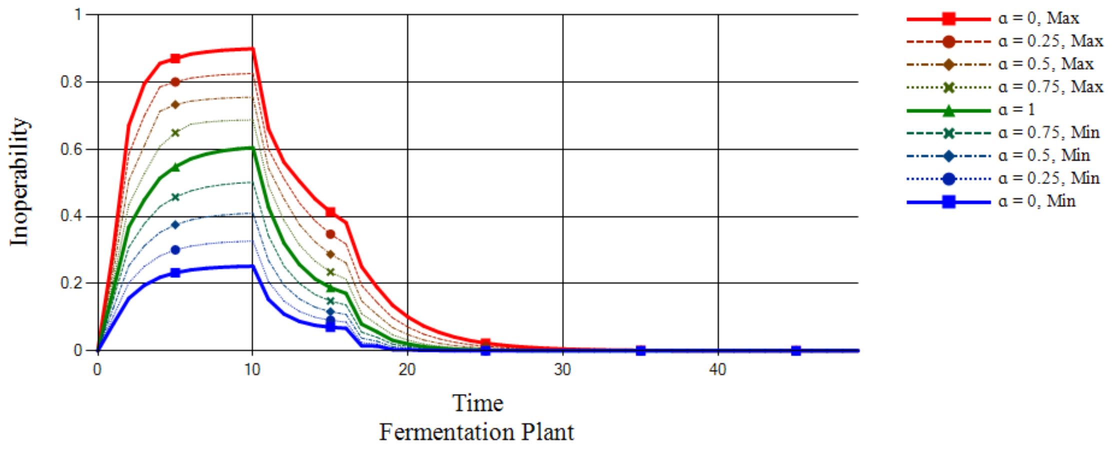

Figure 3- Inoperability timeline of Fermentation Plant in Scenario 2 of Configuration 1

It can be seen that inoperability of the Fermentation Plant is increasing in the first 10 time periods due to a disruption caused by Economic Issues in Region 1 and Insolvency of Supplier 1, after which it gradually decreases until it returns back to full operability in time period 30.

The inoperability timeline of all nodes in the Risk Scenarios 1, 2, 3 and 4 for the both GPN Configurations are shown in Figure 4, Figure 5, Figure 6 and Figure 7, respectively.

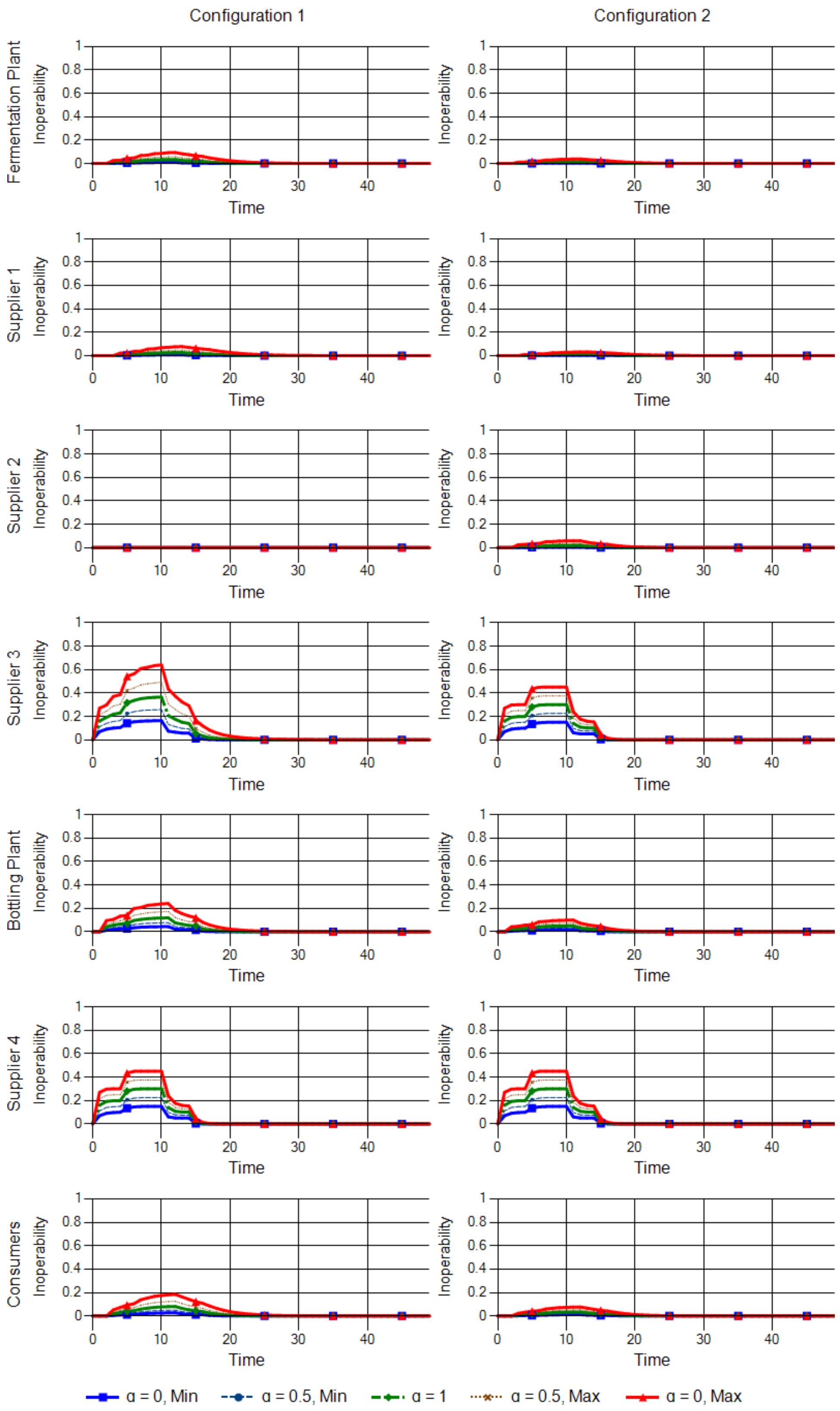

Figure 4- Inoperability of the nodes in both GPN configurations for the Risk Scenario 1

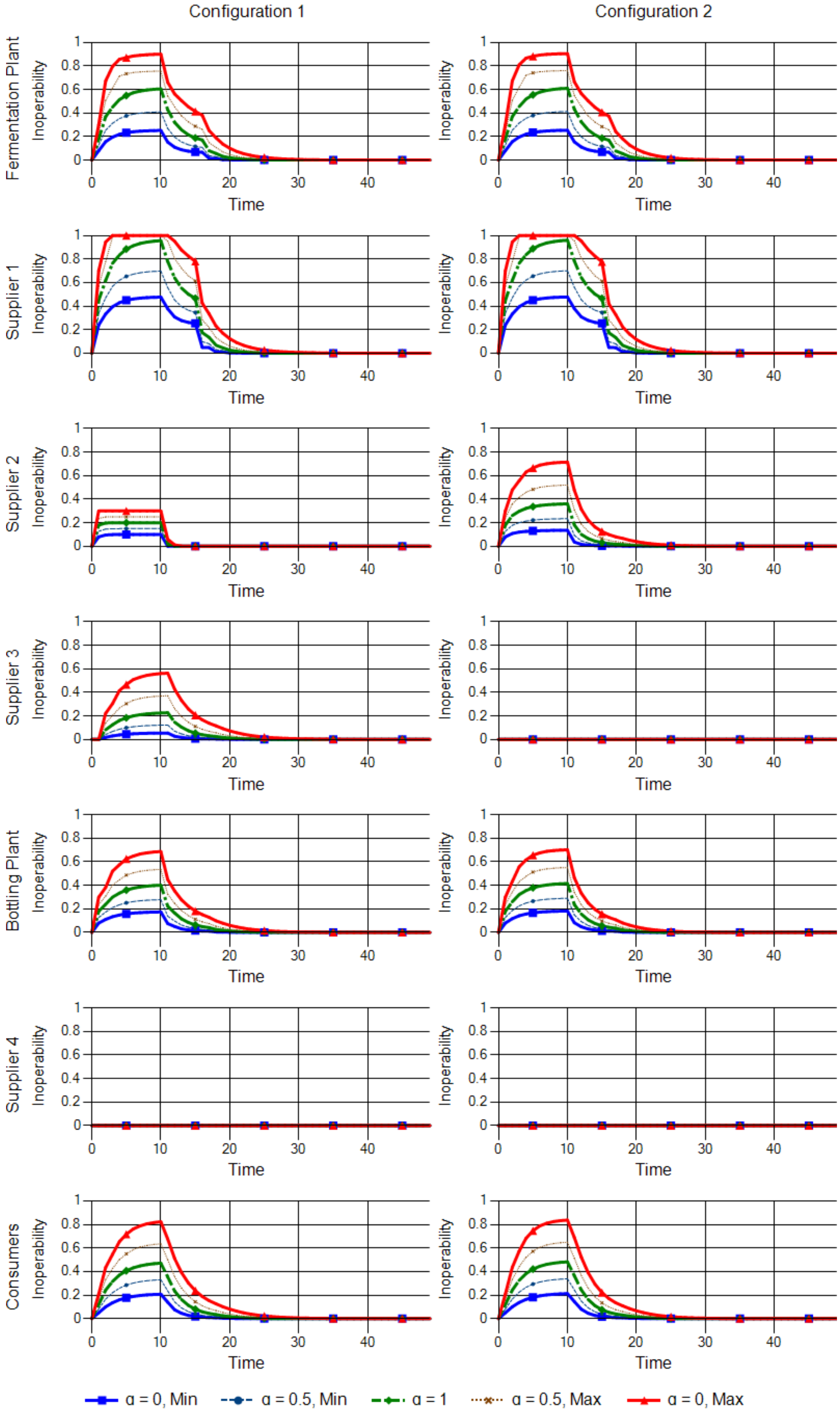

Figure 5- Inoperability of the nodes in both GPN Configurations for the Risk Scenario 2

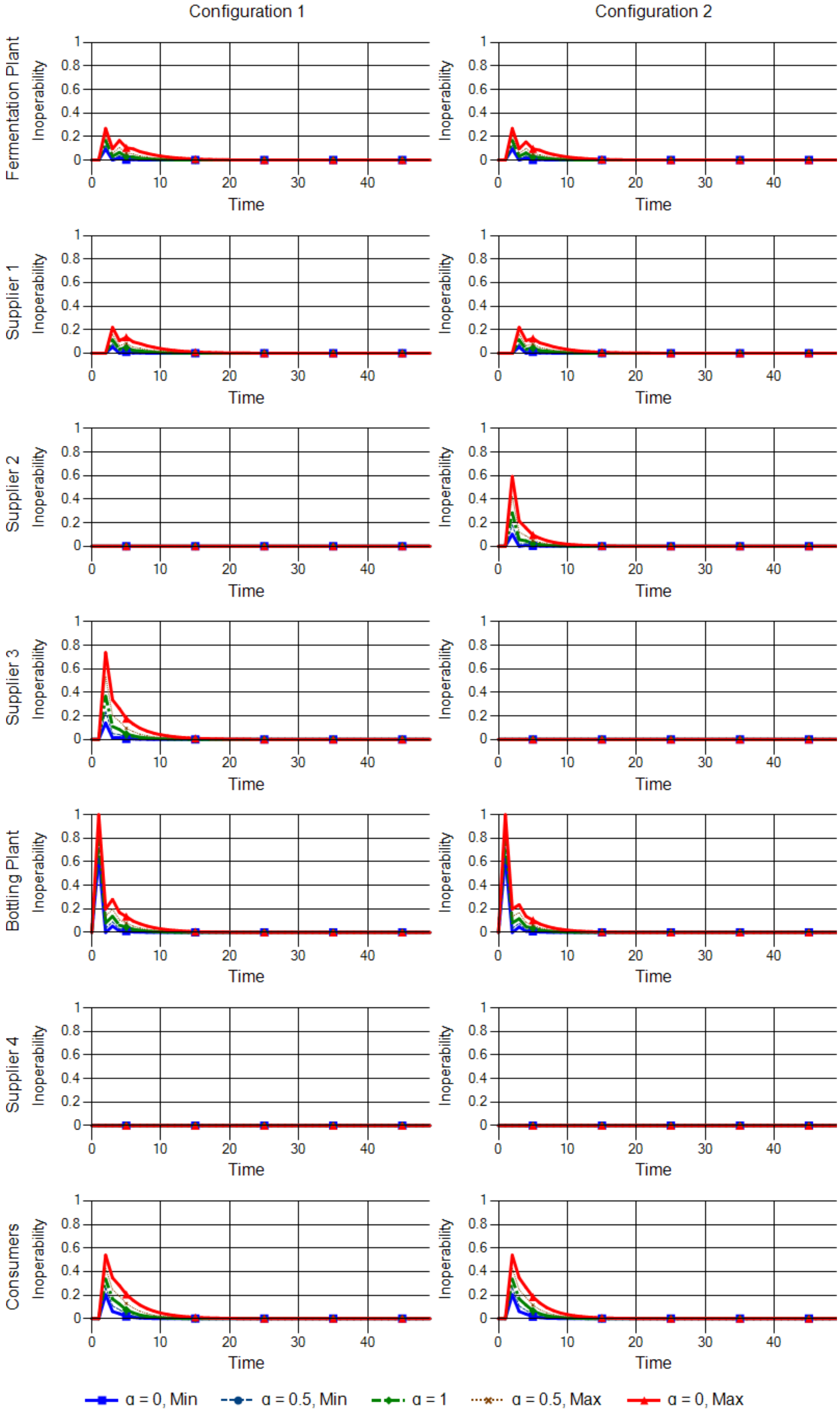

Figure 6- Inoperability of the nodes in both GPN Configurations for the Risk Scenario 3

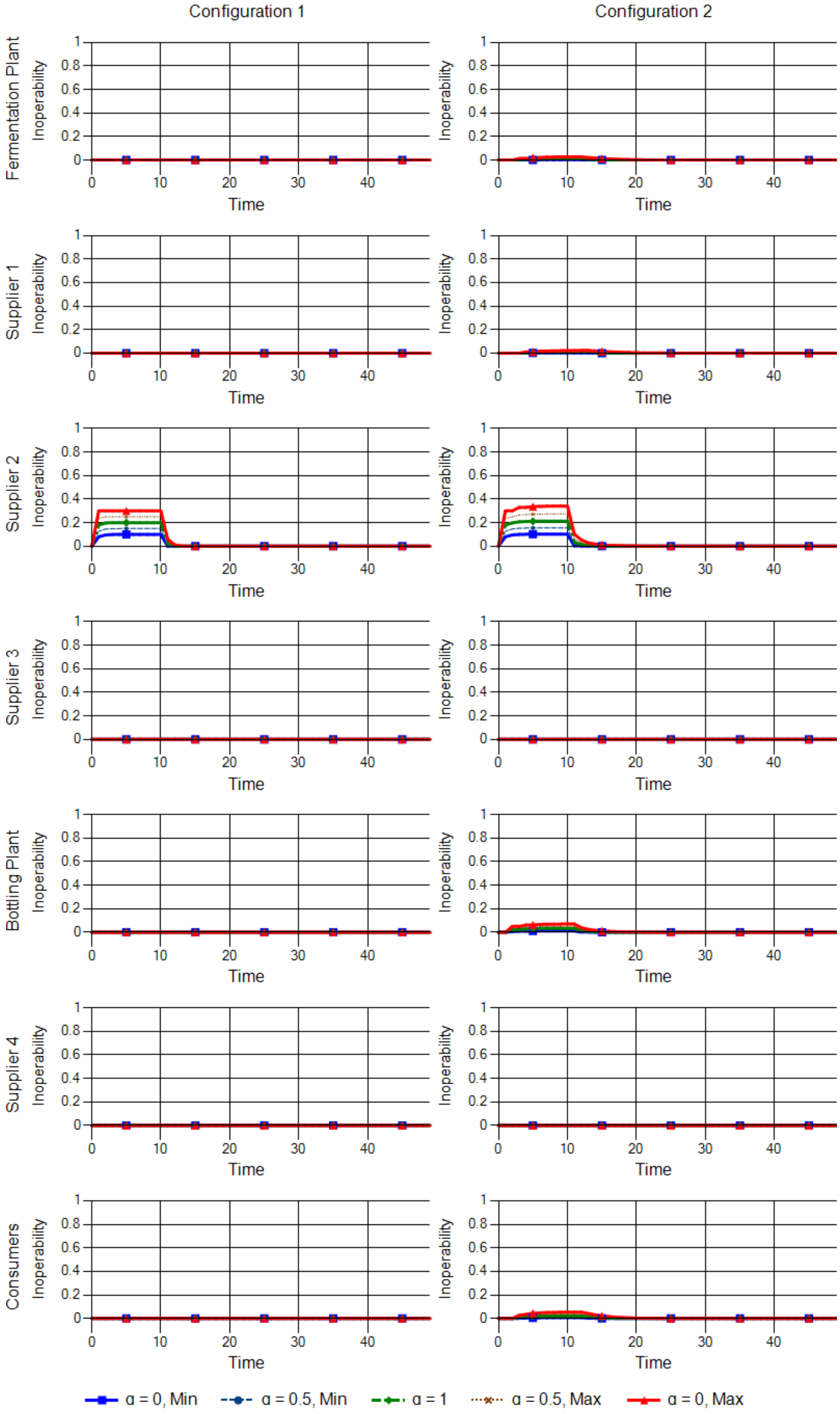

Figure 7- Inoperability of the nodes in both GPN Configurations for the Risk Scenario 4

In Figure 4 (Risk Scenario 1), both Supplier 3 and Supplier 4 in Region 2 are directly affected which results in a major disruption in Configuration 1. In particular, inoperability of Supplier 3 and Supplier 4 propagates to Bottling plant and Consumers, and also, but with a smaller effect on Fermentation and Supplier 1. However, the effects are lower for the Configuration 2 as only Supplier 4 is being utilised. Looking at the trend shown for the Bottling Plant in Configuration 1, the timing of the two perturbations, and the propagation and accumulation of inoperability are quite visible, while, Configuration 2 is affected substantially less than Configuration 1.

In Figure 5 (Risk Scenario 2), Region 1 and Supplier 1 are affected, and that has a substantial impact on both configurations since most of the company's operations are in Region 1 and both configurations depend on Supplier 1. Albeit, inoperabilities of the production facilities are slightly higher in Configuration 2 as Supplier 2 is also located in risk affected Region 1. It is worth mentioning that due to high inoperabilities of Supplier 1, the most likely inoperability ($\alpha = 1$) is closer to the most pessimistic inoperability (upper end point Max, when $\alpha = 0$) than the most optimistic inoperability (lower end point Min, when $\alpha = 0$).

In Figure 6 (Risk Scenario 3), the short-term disruption affects the Bottling Plant and have a similar impact on both configurations and on Supplier 3 and Supplier 2, used exclusively in Configurations 1 and 2, respectively. In this risk scenario, the most likely inoperability values are closer to the most optimistic inoperability, rather than the most pessimistic inoperability. Additionally, a delay can be observed in the inoperability of the adjacent nodes to the Bottling Plant, such as Fermentation Plant, while, even a longer delay is observed for Supplier 1, that is further away from the Bottling Plant.

On the other hand, Scenario 4 in Figure 7 considers a situation where Supplier 2 is struggling with insolvency. As a result, only Configuration 2 is affected. However, due to small interdependency between the Bottling Plant and Supplier 2, the effect is not substantial on other nodes.

The average inoperabilities over the time horizon of all the nodes in the network are estimated as triangular fuzzy numbers for each scenario and both configurations as shown in Table 6. The likelihood of the scenarios are not considered in this calculation.

Table 6- Average inoperability of all the scenarios for both configurations

Average Inoperability	*Configuration 1*	*Configuration 2*
Scenario 1	(0.011, 0.027, 0.053)	(0.009, 0.021, 0.037)
Scenario 2	(0.039, 0.091, 0.157)	(0.039, 0.090, 0.152)
Scenario 3	(0.004, 0.010, 0.023)	(0.004, 0.009, 0.019)
Scenario 4	(0.003, 0.006, 0.009)	(0.003, 0.008, 0.016)
Total	(0.057, 0.134, 0.242)	(0.056, 0.128, 0.224)

As it can be seen from Table 6, the average inoperability in Scenario 1 is higher in the Configuration 1 in comparison with the Configuration 2, which is expected. In Scenario 2, inoperability of Configuration 2 is slightly lower than of Configuration 1, although, the risk affected Region 1 where there are two suppliers in Configuration 2 and only 1 in Configuration 1. This can be justified as in the Configuration 1, Supplier 3 from Region 2 is considerably more affected by the disruption in Region 1 due to high interdependency with the Bottling plant. On the other hand, in Configuration 2, Supplier 3 is not included while Supplier 2 is slightly affected by the disruption in Region 1. This leads to an increased average inoperability in Configuration 1. Furthermore, in Scenario 3, Configuration 2 is again slightly better in terms of inoperability than Configuration 1. This is due to the lower interdependency between Supplier 2 and Bottling plant compared to interdependency between Supplier 3 and Bottling Plant which reduces the overall inoperability. Finally, in Scenario 4, Configuration 2 has higher average inoperability, as Supplier 2 is only used in Configuration 2.

However, inoperability is not zero in Configuration 1, as inoperability of the Supplier 2 is considered in the average inoperabilities of all nodes, despite the fact that it cannot propagate to other nodes.

Another perspective for the comparison of the configurations is the economic effect of risk on the network. Table 7 represents the total loss of risk over the time horizon for both configurations and all the risk scenarios. The expected economic loss of risk for each configuration is calculated by weighting the total economic loss of risk of each scenario by its likelihood using Equation (9).

Table 7- Total economic loss of risk over the time horizon for all the scenarios and the expected economic loss of risk for both configurations

	Economic Loss of Risk	
Scenario	*Configuration 1*	*Configuration 2*
Scenario 1	(1201, 4418, 12456)	(518, 1868, 5079)
Scenario 2	(8370, 26233, 57662)	(8569, 26667, 57687)
Scenario 3	(2007, 4378, 10238)	(1970, 4143, 9161)
Scenario 4	(0, 0, 0)	(321, 1245, 3542)
Expected Economic Loss of Risk	(1141, 5037, 19143)	(1103, 4853, 18196)

Generally, the economic loss of risk is calculated for the nodes that are financially important to the company, which in this case is the Fermentation Plant and Bottling Plant. From Table 7 it can be observed that Scenario 1 is more than twice as costly for the Configuration 1 as opposed to Configuration 2, while average inoperability of Configuration 1 is slightly higher than inoperability of Configuration 2 (presented in Table 2). In this example, Configuration 2 outperforms Configuration 1 in terms of risks, including inoperability and total economic loss of risk. Of course, in practice, in addition to the risk aspect, GPN configurations are also evaluated based on economic measures.

5 Conclusions

In this paper, we investigated the effects and propagation of risk in GPNs at strategic level. Different types of risk factors that affect GPNs, including node specific and regional risk factors, are considered. The concept of risk scenarios for GPNs is introduced to allow for a comparison between various potential GPNs configurations. A novel Fuzzy DIIM is proposed to evaluate the propagation of disruptive events in such networks. Furthermore, a multi criteria method is introduced to determine the interdependencies between nodes in the GPNs. Additionally, the concept of economic loss of risk is utilised to facilitate the comparison of risk effects from the economic aspects. Finally, an illustrative example from food manufacturing is discussed in details to showcase the application of the proposed method. While the provided example is kept simple for illustrative purposes, larger and more complex GPN configuration problems can be studied using the described approach. The large and complex GPN will have more nodes and, consequently, more interdependencies among them. However, the method for determining fuzzy interdependency and the fuzzy DIIM proposed can be easily applied to this type of GPN.

In the future, we will have a closer look at the cost analysis perspective of evaluation of GPN configurations and its application jointly with the risk evaluation proposed. Also, a more granular risk model of GPNs will be developed to be used on the tactical level to be combined with the strategic level risk model.

Acknowledgements

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Appendix A: Fuzzy Arithmetic Definitions

Definition of fuzzy number

A fuzzy number is represented as a fuzzy set over real numbers which can model uncertainty in a value. The fuzzy set determines the membership degree of any real number to the fuzzy number; a membership degree of 1 shows the complete membership, while a degree of 0 shows no membership and degrees between 0 and 1 present a partial membership of a number to the fuzzy value. The function that determines the membership degree of a real number to the fuzzy number is referred to as the membership function. Fuzzy numbers are assumed to have a piece-wise continuous and convex membership functions and are normalised, i.e., have exactly one point with a membership degree of one (Klimke, 2006).

Definition of triangular fuzzy number

Triangular fuzzy numbers are one of the most often used types of fuzzy numbers that are identified by a triplet: $\tilde{X} = (X_1, X_2, X_3)$ where $X_1 \leq X_2 \leq X_3$, where X_1 is the lower boundary with membership degree 0, X_2 is the modal value with membership degree 1 and X_3 is the upper boundary with membership degree 0, of possible values that the fuzzy number can take. The membership degrees between these points are determined by linear functions.

Definition of Extension principle

Extension principle, introduced by Zadeh (1975), is a crucial method which defines the way fuzzy calculations are performed on fuzzy numbers. This principle defines that for any function, the membership degree of the output of function is the supremum of the minimum memberships of all input values to their corresponding fuzzy sets that will result in the output value; or in other words, for function $f: S_1 \times S_2 \times \ldots \times S_n \rightarrow S$ where S_1, S_2, …, S_n and S are sets of real numbers, and fuzzy sets $\widetilde{X_1} \subseteq S_1, \widetilde{X_2} \subseteq S_2, \ldots, \widetilde{X_n} \subseteq S_n$:

$$\mu_{f(\widetilde{X_1},\widetilde{X_2},\ldots,\widetilde{X_n})}(y) = \sup_{x_1 \in \widetilde{X_1},\ldots,x_n \in \widetilde{X_n} \wedge\, y=f(x_1,x_2,\ldots,x_n)} \min\left[\mu_{\widetilde{X_1}}(x_1), \mu_{\widetilde{X_2}}(x_2), \ldots, \mu_{\widetilde{X_n}}(x_n)\right]$$

However, Extension principle is computationally prohibitive. More efficient calculations methods have been proposed. Fuzzy addition, unary negation, and scalar multiplication of triangular fuzzy numbers $\tilde{X} = (X_1, X_2, X_3)$ and $\tilde{Y} = (Y_1, Y_2, Y_3)$ and real non-negative scalar value β, which are used in the method proposed, can be efficiently calculated as follows:

1) $\tilde{X} + \tilde{Y} = (X_1 + Y_1, X_2 + Y_2, X_3 + Y_3)$
2) $-\tilde{X} = (-X_3, -X_2, -X_1)$
3) $\beta\tilde{X} = (\beta X_1, \beta X_2, \beta X_3)$

Definition of α-cut

An α-cut of a fuzzy number (or fuzzy set) is a crisp set of values that have a membership degree of at least α. The membership function of a fuzzy number is assumed to be convex, which means that any α-cut of fuzzy numbers is an interval that can be identified by its lower and upper endpoints, X^{L_α} and X^{U_α}, respectively. Fuzzy calculations can be simplified by discretising α and using the interval calculations to determine the corresponding endpoints of α-cuts. This approach is used in the fuzzy DIIM proposed.

An α-cut of the triangular fuzzy number $\tilde{X}$ is illustrated in Figure 8.

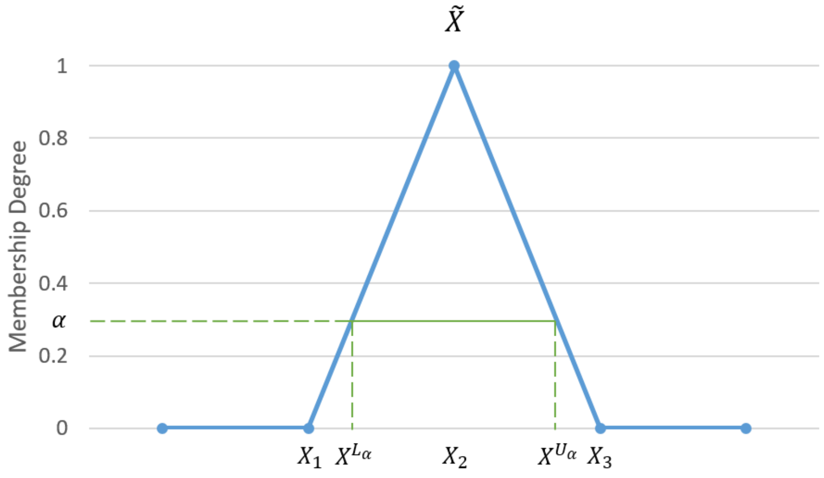

Figure 8- An α-cut of a triangular fuzzy number

Appendix B: Fuzzy DIIM

The basic arithmetic operations on triangular fuzzy numbers are simple. However, while triangular fuzzy numbers are closed for addition and subtraction, they are not closed for multiplication and division. Different procedures which approximate results of multiplication and division and express them as triangular fuzzy numbers have been proposed in the literature (Giachetti and Young, 1997). As Equation (7) includes multiple multiplications and is applied iteratively, we developed an exact method for fuzzy arithmetic in the fuzzy DIIM method based on α-cuts. As discussed in Section 3.5, a vector representation of the fuzzy DIIM proposed can be written as follows:

$$\tilde{q}(t+1) = \tilde{K}\widetilde{A^*}\tilde{q}(t) + \tilde{K}\widetilde{c^*}(t) + \left(I - \tilde{K}\right)\tilde{q}(t)$$

A non-vector version of this model is as follows:

$$\tilde{q_i}(t+1) = \widetilde{K_{i,i}} \sum_j \widetilde{A_{i,j}^*}\tilde{q_j}(t) + \widetilde{K_{i,i}}\widetilde{c_i^*}(t) + \left(1 - \widetilde{K_{i,i}}\right)\tilde{q_i}(t)$$

and a crisp non-vector representation can be written as:

$$q_i(t+1) = K_{i,i} \sum_j A_{i,j}^* q_j(t) + K_{i,i}c_i^*(t) + \left(1 - K_{i,i}\right)q_i(t) \tag{10}$$

Since this function is continuous on all variables and parameters, the calculation can be simplified to finding the output $q_i(t)$ for each α-cut, $\alpha \in [0,1]$), which means effectively determining a lower $q_i^{L\alpha}(t)$ and an upper $q_i^{U\alpha}(t)$ endpoints. For a general function, this is equivalent to two global optimisation problems which have to find the global minimum and maximum values of the function in the boundary space determined by the α-cut intervals of the fuzzy input variables and parameters. We are analysing and applying an efficient method to determine the global optima for function (10) as follows.

We need to find the stationary points of function (10) which requires the partial differentials on one of the variables or parameters to be zero. We investigate this requirement, for all variables and parameters in function (10), as follows:

1) $\dfrac{\partial q_i(t+1)}{\partial q_i(t)} = K_{i,i}A_{i,i}^{*} + \left(1 - K_{i,i}\right) > 0$

2) $\dfrac{\partial q_i(t+1)}{\partial q_j(t)} = K_{i,i}A_{i,j}^{*} > 0; \qquad \text{for all } j \neq i$

3) $\dfrac{\partial q_i(t+1)}{\partial A_{i,j}^{*}} = K_{i,i}q_j(t) > 0; \qquad \text{for all } j$

4) $\dfrac{\partial q_i(t+1)}{\partial c_i^{*}(t)} = K_{i,i} > 0$

5) $\dfrac{\partial q_i(t+1)}{\partial K_{i,i}} = \sum_j A_{i,j}^{*}q_j(t) + c_i^{*}(t) - q_i(t)$

The function (10) is continuous and all partial differentials (1) to (4), except for the resilience matrix (5), are positive which implies that $q_i(t+1)$ is increasing when $q_i(t)$, $q_j(t)$, $A_{i,j}^{*}$ and $c_i^{*}(t)$ are increasing. Therefore, assuming that $K_{i,i}$ is fixed, the minimum (maximum) value of function (10) ${q_i}^{L\alpha}(t+1)$ (and ${q_i}^{U\alpha}(t+1)$) in an α-cut interval is obtained by using the minimum (maximum) values ${q_i}^{L\alpha}(t)$, ${q_j}^{L\alpha}(t)$, ${A_{i,j}^*}^{L\alpha}$ and ${c_i^*}^{L\alpha}(t)$ (${q_i}^{U\alpha}(t)$, ${q_j}^{U\alpha}(t)$, ${A_{i,j}^*}^{U\alpha}$ and ${c_i^*}^{U\alpha}(t)$). For fixed value $K_{i,i}=k$, the following holds:

$$\begin{aligned} q_i^{L\alpha}(t+1) &= k\sum_j {A_{i,j}^*}^{L\alpha} q_j^{L\alpha}(t) + k{c_i^*}^{L\alpha}(t) + (1-k)q_i^{L\alpha}(t) \\ &= k\left[\sum_j {A_{i,j}^*}^{L\alpha} q_j^{L\alpha}(t) + {c_i^*}^{L\alpha}(t) - q_i^{L\alpha}(t)\right] + q_i^{L\alpha}(t) \end{aligned} \tag{11}$$

$$\begin{aligned} q_i^{U\alpha}(t+1) &= k\sum_j {A_{i,j}^*}^{U\alpha} q_j^{U\alpha}(t) + k{c_i^*}^{U\alpha}(t) + (1-k)q_i^{U\alpha}(t) \\ &= k\left[\sum_j {A_{i,j}^*}^{U\alpha} q_j^{U\alpha}(t) + {c_i^*}^{U\alpha}(t) - q_i^{U\alpha}(t)\right] + q_i^{U\alpha}(t) \end{aligned} \tag{12}$$

The question is which value of $K_{i,i}$ leads to the minimum value, ${q_i}^{L\alpha}(t+1)$, and the maximum value, ${q_i}^{U\alpha}(t+1)$, of function (10).

If $\sum_j {A_{i,j}^*}^{L\alpha} q_j^{L\alpha}(t) + {c_i^*}^{L\alpha}(t) - q_i^{L\alpha}(t) \geq 0$ then (11) is increasing in the α-cut of $\widetilde{K_{i,i}}$ from $K_{i,i}^{L\alpha}$ to $K_{i,i}^{U\alpha}$ and:

$$q_i(t+1) \geq K_{i,i}^{L\alpha}\left[\sum_j {A_{i,j}^*}^{L\alpha} q_j^{L\alpha}(t) + {c_i^*}^{L\alpha}(t) - q_i^{L\alpha}(t)\right] + q_i^{L\alpha}(t)$$

This implies that minimum of $q_i(t+1)$, $q_i^{L\alpha}(t+1)$ is achieved for $K_{i,i}^{L\alpha}$.

Also, if $\sum_j {A_{i,j}^*}^{U\alpha} q_j^{U\alpha}(t) + {c_i^*}^{U\alpha}(t) - q_i^{U\alpha}(t) \geq 0$ then (12) is increasing in the α-cut of $\widetilde{K_{i,i}}$ from $K_{i,i}^{L\alpha}$ to $K_{i,i}^{U\alpha}$ and:

$$q_i(t+1) \leq K_{i,i}^{U\alpha}\left[\sum_j {A_{i,j}^*}^{U\alpha} q_j^{U\alpha}(t) + {c_i^*}^{U\alpha}(t) - q_i^{U\alpha}(t)\right] + q_i^{U\alpha}(t)$$

This implies that maximum of $q_i(t+1)$, $q_i^{U\alpha}(t+1)$ is achieved for $K_{i,i}^{U\alpha}$.

If $\sum_j {A^*_{i,j}}^{L\alpha} q_j^{L\alpha}(t) + {c_i^*}^{L\alpha}(t) - q_i^{L\alpha}(t) < 0$ then (11) is decreasing in the α-cut of $\widetilde{K_{\iota,\iota}}$ from $K_{i,i}^{L\alpha}$ to $K_{i,i}^{U\alpha}$ and:

$$q_i(t+1) \geq \; K_{i,i}^{U\alpha}\left[\sum_j {A^*_{i,j}}^{L\alpha} q_j^{L\alpha}(t) + {c_i^*}^{L\alpha}(t) - q_i^{L\alpha}(t)\right] + q_i^{L\alpha}(t)$$

This implies that minimum of $q_i(t+1)$, $q_i^{L\alpha}(t+1)$ is achieved for $K_{i,i}^{U\alpha}$.

Also, if $\sum_j {A^*_{i,j}}^{U\alpha} q_j^{U\alpha}(t) + {c_i^*}^{U\alpha}(t) - q_i^{U\alpha}(t) < 0$ then (12) is decreasing in the α-cut of $\widetilde{K_{\iota,\iota}}$ from $K_{i,i}^{L\alpha}$ to $K_{i,i}^{U\alpha}$ and:

$$q_i(t+1) \leq \; K_{i,i}^{L\alpha}\left[\sum_j {A^*_{i,j}}^{U\alpha} q_j^{U\alpha}(t) + {c_i^*}^{U\alpha}(t) - q_i^{U\alpha}(t)\right] + q_i^{U\alpha}(t)$$

This implies that maximum of $q_i(t+1)$, $q_i^{U\alpha}(t+1)$ is achieved for $K_{i,i}^{L\alpha}$.

<u>Calculation procedure for fuzzy DIIM</u>

The inoperability is determined by finding α-cut intervals that represent the inoperability value at various membership degrees $\alpha \in [0,1]$. The lower and upper endpoints of the α-cut interval for inoperability of node i at time period t for membership degree α, $q_i^{\mathrm{L}\alpha}(t)$ and $q_i^{\mathrm{U}\alpha}(t)$ respectively, are calculated as follows:

From $\mathrm{t} = 0$ to all time periods in the time horizon.

From $i = 1$, to all the nodes in the network.

For $\alpha = 0$ to $\alpha = 1$ with an arbitrary step increment.

Beginning

Step 1: If $\sum_j A_{i,j}^{*\mathrm{L}\alpha} q_j^{\mathrm{L}\alpha}(t) + {c_i^*}^{\mathrm{L}\alpha}(t) \geq q_i^{\mathrm{L}\alpha}(t)$ then K'_{min}=$K_{i,i}^{\mathrm{L}\alpha}$

Else K'_{min}=$K_{i,i}^{\mathrm{U}\alpha}$.

Step 2: If $\sum_j A_{i,j}^{*\mathrm{U}\alpha} q_j^{\mathrm{U}\alpha}(t) + c_i^{*\mathrm{U}\alpha}(t) \geq q_i^{\mathrm{U}\alpha}(t)$ then K'_{max}=$K_{i,i}^{\mathrm{U}\alpha}$

Else K'_{max}=$K_{i,i}^{\mathrm{L}\alpha}$.

Step 3: $q_i^{\mathrm{L}\alpha}(t+1) = K'_{min} \sum_j A_{i,j}^{*\mathrm{L}\alpha} q_j^{\mathrm{L}\alpha}(t) + K'_{min} c_i^{*\mathrm{L}\alpha}(t) + (1 - K'_{min}) q_i^{\mathrm{L}\alpha}(t)$.

Step 4: $q_i^{\mathrm{U}\alpha}(t+1) = K'_{max} \sum_j A_{i,j}^{*\mathrm{U}\alpha} q_j^{\mathrm{U}\alpha}(t) + K'_{max} c_i^{*\mathrm{U}\alpha}(t) + (1 - K'_{max}) q_i^{\mathrm{U}\alpha}(t)$.

End

where $A_{i,j}^{*\mathrm{L}\alpha}$, $c_i^{*\mathrm{L}\alpha}(t)$ and $K_{i,i}^{\mathrm{L}\alpha}$ ($A_{i,j}^{*\mathrm{U}\alpha}$, $c_i^{*\mathrm{U}\alpha}(t)$ and $K_{i,i}^{\mathrm{U}\alpha}$) are the lower (upper) end-points of the fuzzy interdependency value between nodes i and j, the fuzzy perturbation level of nodes i at time period t and the fuzzy resilience of node i, respectively, while, K'_{min} and K'_{max} are the resilience values of node i that lead to the minimum and maximum value of inoperability, respectively.

Steps 1 and 2 find the corresponding resilience values in the α-cut of $K_{i,i}$, i.e., interval $\left[K_{i,i}^{\mathrm{L}\alpha}, K_{i,i}^{\mathrm{U}\alpha}\right]$ that can yield the lower and upper endpoint values of the α-cut of inoperability $q_i^{\mathrm{L}\alpha}(t+1)$ and $q_i^{\mathrm{U}\alpha}(t+1)$, respectively, following rules of multiplication of α-cuts of fuzzy numbers. One can notice that depending on the value of resilience's coefficients, resilience can have either a direct impact on inoperability when K'_{min}=$K_{i,i}^{\mathrm{L}\alpha}$ and K'_{max}=$K_{i,i}^{\mathrm{U}\alpha}$ or an inverse impact when K'_{min}=$K_{i,i}^{\mathrm{U}\alpha}$ and K'_{max}=$K_{i,i}^{\mathrm{L}\alpha}$.

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